Microwave Imaging: an Iterative Numerical Solution

N. Joachimowicz, A. Franchois, Ch. Pichot, A. Joisel

Laboratoire des Signaux et Systèmes (CNRS/ESE)
Plateau de Moulon, 91192 Gif-sur-Yvette, France

J.Ch. Bolomey, J.P. Hugonin, L. Garnero

Institut d'Optique, Bâtiment 503, 91405 Orsay, France

ABSTRACT

This paper deals with an iterative method for solving an inverse electromagnetic scattering problem: the quantitative reconstruction of complex permettivity distribution of inhomogeneous 2D or 3D lossy dielectric objects from measured scattered near field data.
Using an exact integral equation and the Moment method, matrix equations are obtained for the forward scattering problem. The inverse problem is solved by an iterative procedure based on a Newton-Kantorovich method. The solution has been constructed for different polarization cases (2D-TM, 2D-TE and 3D) and in order to incorporate multi-incidence configuration. A Tikhonov regularization is applied on ill-conditioned matrices which have to be inverted.
This iterative numerical solution provides quantitative reconstruction of the complex permittivity profile even for strong scatterers as encountered in biomedical applications.
Numerical result are presented showing the convergence of solution and the influence of noise is also investigated on 3D object in order to test the robustness of the algorithm.

INTRODUCTION

This paper deals with an inverse scattering problem: the reconstruction of complex permittivity of lossy dielectric objects from scattered near-field measurements.
During the last decade, an increasing interest has been devoted to the determination of the complex permittivity profile of 2D or 3D objects from moment method solutions of the integral equations (Hagmann et al., 1981), (Ghodgaonkar et al., 1983), (Johnson and Tracy, 1983), (Ney et al. 1984), (Datta and Bandyopadhyay, 1986), (Guo and Guo, 1987), (Caorsi et al., 1988 and 1990). At present, they appear to be among the most promising approaches for Microwave Imaging. Nevertheless the stability of such approaches is very sensitive to the observation point locations and measurement accuracy due to the nature of the inverse problem, which is in general strongly nonlinear when quantitative imaging is requested, and ill-posed. This involves, in general, the use of a regularization procedure. An iterative solution can have some important advantages: effects of ill-conditioning can be significantly reduced by enforcing the convergence with *a priori* information.
The iterative numerical solution has been constructed in order to incorporate multi-incidence configuration (the object is successively illuminated by different incident fields). Different codes involving the implementation of forward and inverse problems for 2D and 3D cases have been developed. Reconstruction in the presence of noise is presented in order to test the robustness of the algorithm.

1. FORMULATION

Let an incident field with electric field $\mathbf{E}^i$ illuminate an inhomogeneous dielectric object of complex permittivity $\varepsilon*(\mathbf{r})$ and arbitrary shape defining a volumic domain D surrounded by a homogeneous medium of complex permettivity ε_1*. The object being illuminated by a known incident field, the scattered field E^s is collected by M detectors located around the object or in a defined region close to the object. We use the Moment method applied to an Electric Field Integral Equation (EFIE):

$$\mathbf{E}^S(\mathbf{r}) = \iiint_D C(\mathbf{r}')\,\mathbf{E}(\mathbf{r}')\,{}^d G(\mathbf{r},\mathbf{r}')\,d\mathbf{r}' \tag{1}$$

The kernel or tensor Green's function ${}^d G(\mathbf{r},\mathbf{r}')$ (for 2D-TM, 2D-TE and 3D cases) is defined as:

$${}^1 G(\mathbf{r},\mathbf{r}') = i/4\, H_0^1(k_1 \,|\, \mathbf{r}-\mathbf{r}' \,|) \qquad \text{for 2D-TM case (2a)}$$

$${}^2 G(\mathbf{r},\mathbf{r}') = (I + 1/k_1^2\, \nabla\,.\,\nabla)\, i/4\, H_0^1(k_1 \,|\, \mathbf{r}-\mathbf{r}' \,|) \qquad \text{for 2D-TE case (2b)}$$

$^3G(\mathbf{r}, \mathbf{r}') = (I + 1/k_1^2 \nabla.\nabla) \exp(i k_1 |\mathbf{r}-\mathbf{r}'|)/(4\pi|\mathbf{r}-\mathbf{r}'|)$ for 3D case (2c)

(for a time dependance in $e^{-i\omega t}$) where $H_0^{(1)}$ stands for the Hankel function of zero order of the first kind and the function $C(\mathbf{r}')$ represents the unknown contrast between the object and the background medium:

$$C(\mathbf{r}') = k_{0bj}^2(\mathbf{r}') - k_1^2 \quad (3)$$

In case of multiple incidence configuration, the object is illuminated successively by N_v different angles of incidence.

The Moment method leads to the following matrix relation:

$$[E^s] = [G][C][E] \quad (4)$$

where:

- [C] represents a dNxdN diagonal matrix whose elements depend on the local permittivity contrast of the object.
- [G] represents a dMxdN matrix whose elements are calculated from expression of the Green's function.
- [E] represents a dN vector whose components are the values of the total field at the sampling points.
- $[E^s]$ represents a dM vector whose components are the scattered field at the collected points.

The former matrix relation (4) is extended to V scattered fields E^s collected for V different angles of the incident field. Following the iterative method developed by (Joachimowicz et al., 1991), the variation ΔE^s of E^s (dimension VM) induced by a small variation of ΔC is given by:

$$[\Delta E^s] = [D][\Delta C] \quad (5)$$

where [D] is a VdmxN matrix and $[\Delta E^s]$ is a VdM vector. The inversion of the ill-conditioned matrix [D] using a Tikhonov regularization procedure (Tikhonov and Arsenine, 1977), provides the contrast error $[\Delta C]$ for updating the initial guess:

$$[\Delta C] = (D^* D + \alpha I)^{-1} D^* [\Delta E^s] \quad (6)$$

Starting from an initial guess of the permittivity distribution in the object, the computed scattered field E^s is compared to the measurement at each step of the iteration. The iterative process is stopped when $[\Delta E^s]$ is close enough to zero.

The domain of validity of this method is directly related to the signal to noise ratio. Stability sensitivity is why the use of an iterative scheme is important: effects of ill-conditioning can be significantly reduced by enforcing the convergence with a priori information (object external shape, upper and lower bounds of complex permittivity, presence of different media,...).

2. RESULTS

The method is illustrated with various numerical simulations of practical interest performed on dissipative inhomogeneous dielectric objects (biological phantoms, human arm,...) in view of biomedical applications of microwave imaging. The influence of noise has also been studied in order to test the robustness of the algorithm. The numerical simulations have been performed on a VAX 8300 computer. The objects have dielectric properties close to those of biological tissues (bone, muscle, fat) and are immersed in water, as it is usual for such applications. At 3 GHz the different chosen relative complex permittivities are $\varepsilon_r^* = (8., 1.2)$ for bone, $\varepsilon_r^* = (46., 12)$ for muscle (Joachimowicz et al., 1991) and $\varepsilon_r^* = (76., 14.4)$ for water.

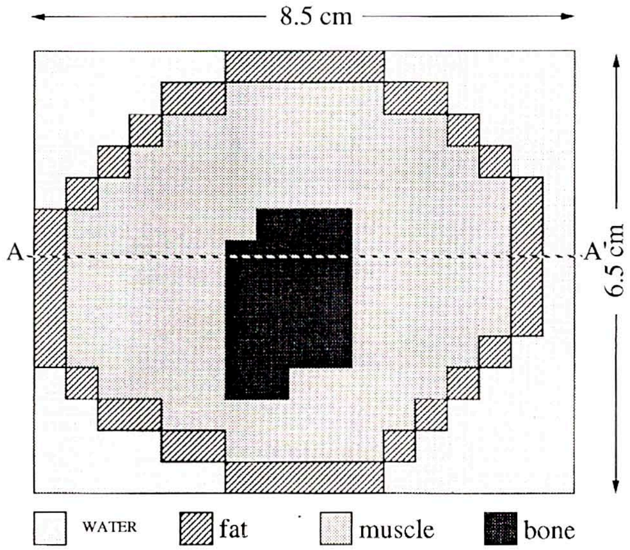

Fig. 1 - Numerical model of a human arm cross-section.

One considers a human arm cross-section as shown in Fig.1. The arm section is enclosed inside a rectangular grid which is divided into 221 0.43λ-sided square cells. The number of TM-polarized plane waves used is 32. For each incident plane wave, the scattered field is measured with 32 receivers located on a circle of radius 5λ. The initial guess corresponds to a rectangular grid composed of homogeneous muscle. Fig. 2a and 2b show the convergence of the reconstruction after 14 iterations on the complex permittivity (real and imaginary part, respectively) when a piece of bone is introduced in the initial guess. As can be seen, the convergence is achieved in a complicated configuration. Moreover, similar results have been obtained in the TE case (Joachimowicz et al., 1991) which is a more difficult case than the TM one according to the vectorial aspect of the scattering problem.

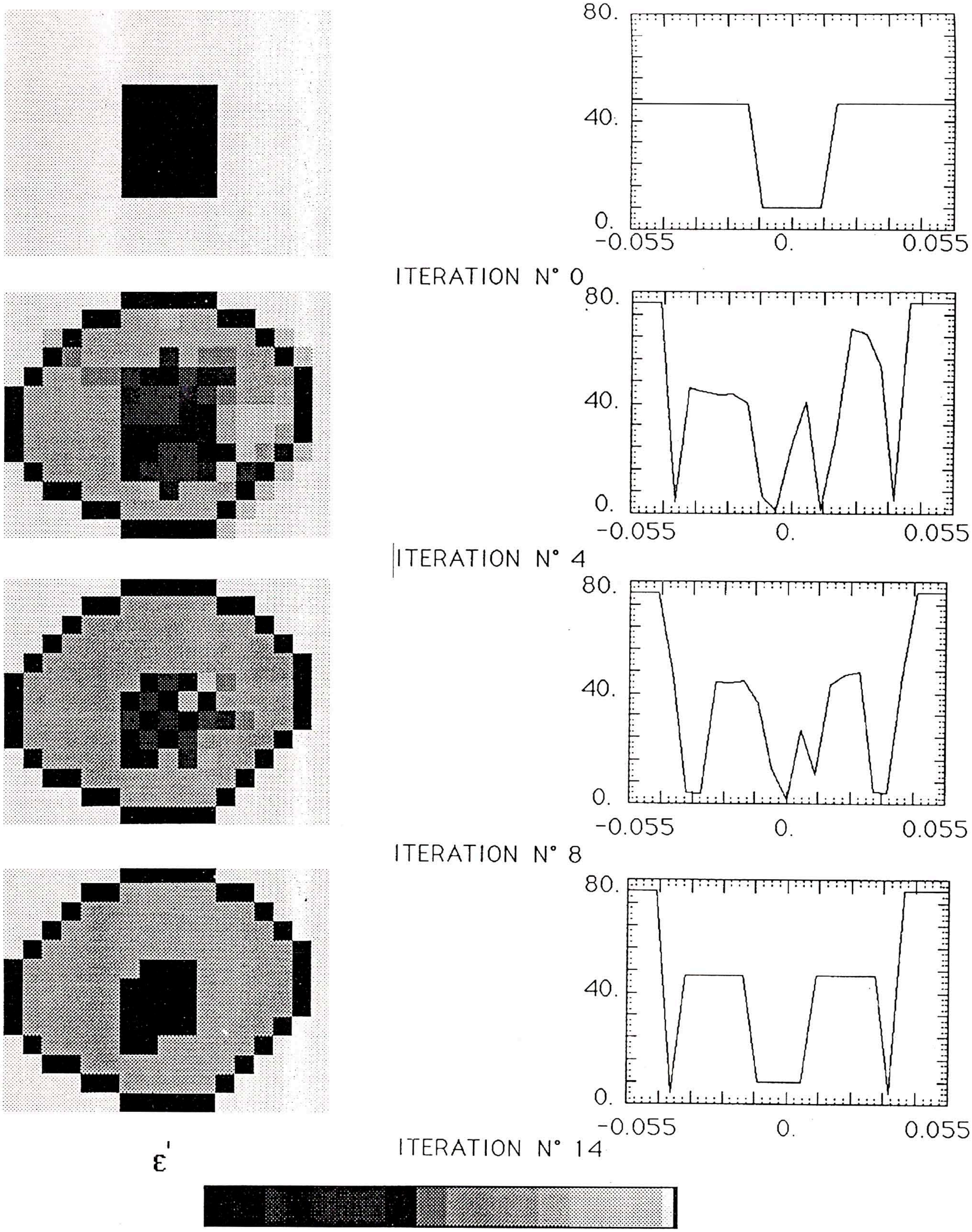

Fig. 2a - Reconstruction of the real part of the complex permittivity distribution at 3 GHz with 32 plane waves in the TM polarization case. Reconstructed image (left) and cross cuts along the profile AA' (right).

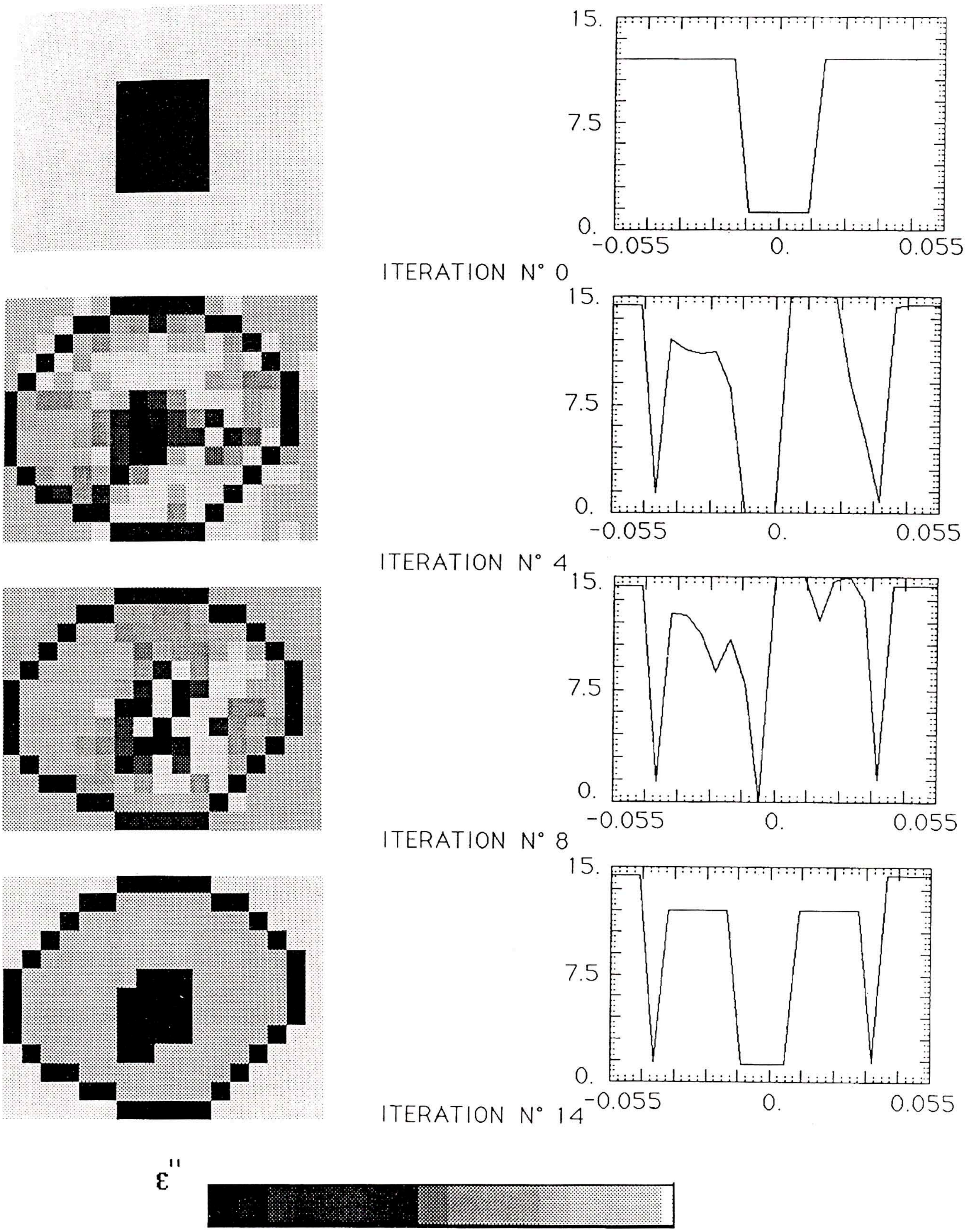

Fig. 2b - Reconstruction of the imaginary part of the complex permittivity distribution at 3 GHz with 32 plane waves in the TM polarization case. Reconstructed image (left) and cross cuts along the profile AA' (right).

The relative mean square error ERR_c of the reconstructed permittivity profile id defined as:

$$ERR_c = \left[\sum_{i=1}^{N} |\Delta c_k(i)|^2 / \sum_{i=1}^{N} |C(i)|^2 \right]^{\frac{1}{2}} \quad (7)$$

where i, C and Δ c_k denote, respectively, the number of the cell, the exact value of the contrast and the difference between the reconstructed and the exact value at the k-th step.

Figura 3 illustrates the effect of *a priori* knowledge on the convergence rate. It appears that CPU time can twice be reduced by choosing an appropriate initial guess closer to the real configuration.

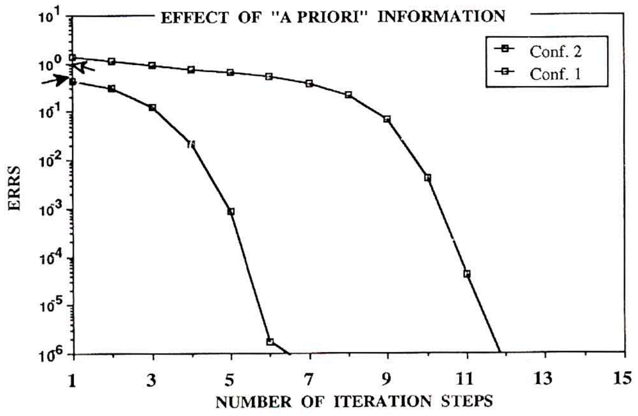

Fig. 3 - Effect of a priori information on the convergence of ERR_c in the TM case; configuration n° 1: the initial guess is a piece of bone surrounded by muscle; configuration n° 2: the initial guess with the piece of bone and external contour.

According to these results, the spatial iterative technique presented here provides more accurate and useful reconstruction as well as more flexibility in considering different experimental arrangements, arbitrary polarization case and *a priori* information. Such performance partly results in an incrase of the computation time.

For 3D reconstruction, Figura 4 shows the geometry of the problem and the related 3D representation used to depict the results. A dielectric cube divided into 27 cells, whose side is equal to (λ) (wavelength in the free space) has been investigated. In this example, the cubic volume contains an inhomogeneity (one cell) of complex permittivity $\varepsilon_{0\,bj}^* = (3., 0.)$. The cube is illuminated by 32 plane waves and 12 receivers are located on 4 parallel lines located on each side of the dielectric object, at (λ) from the center (planar receiver geometry). The distance between two receivers on each line is equal to $\lambda/3$.

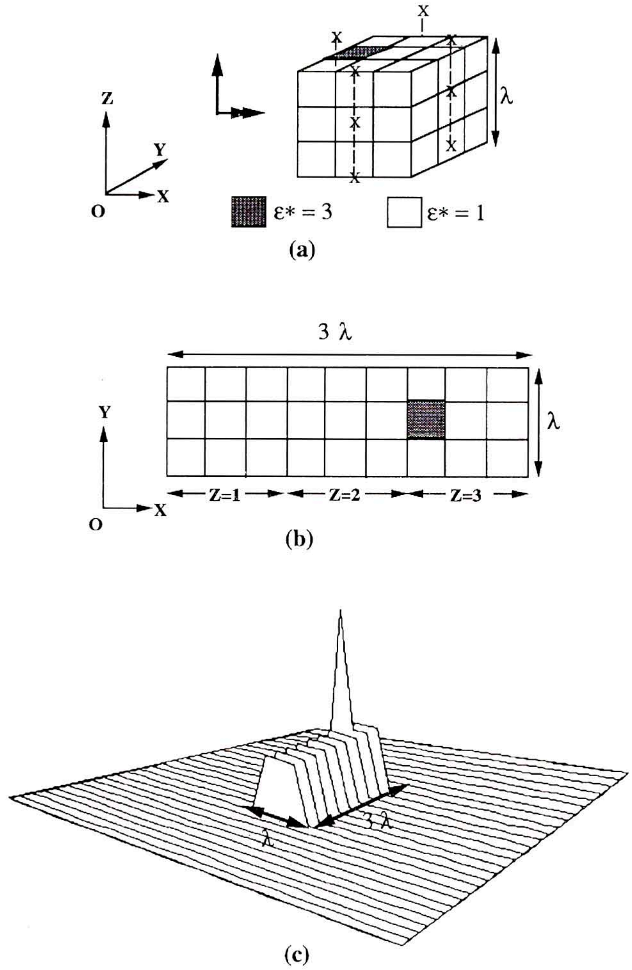

Fig. 4 - The three-layered investigation domain and its 3D representation.

The noise effect has been studied by adding a random measurement error, characterized by a uniform distribution with a maximum precent magnitude w. Fig.13 a, b, c respectively show the initial guess (a free space cube) and the results obtained after 3 iterations, for w=5 and w=20. As we can see, the peak distinctly appears, for both cases.

CONCLUSION

A numerical solution for solving microwave imaging problems has been proposed, for the complex permittivity reconstruction of inhomogeneous dielectric objects from near-field measurements. 2D-TM as well as 2D-TE and 3D cases have been investigated and carried out.

The spatial technique presented here provides quantitative imaging even with strong diffraction effects. Consequently, this method seems to offer more flexibility in considering *a priori* information, different polarization cases and multi-incidence configuration.

The influence of noise has also been studied in order to

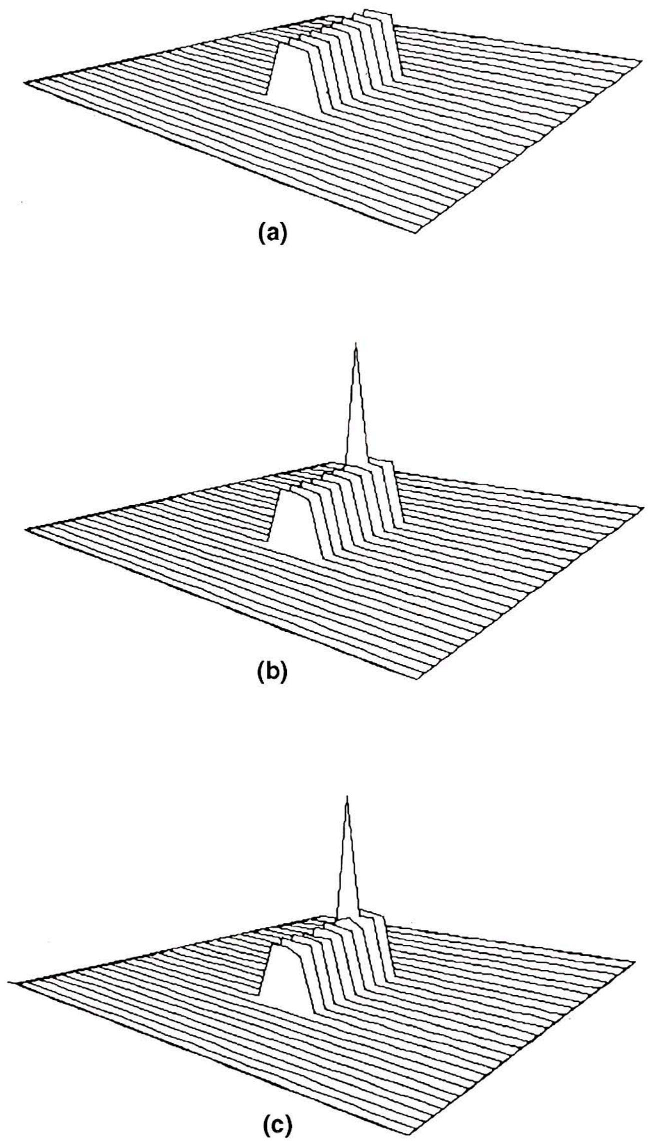

Fig. 5 - Representation of the amplitude of the complex permittivity. a) initial profile, b) reconstruction with w=5, c) reconstruction with w=20.

test the robustness of the algorithm and the solution seems to be relatively robust in terms of signal to noise ratio. The method of Generalized Cross Validation seems to be a good attempt for choosing the regularization parameter specially with noisy data (Franchois and Pichot, 1992).

As a result of the possibility of including in the iterative procedure, multiple view configuration and *a priori* information, this technique appears a promising attempt to overcome the limitations of conventional diffraction effects.

First encouraging reconstruction have been obtained using real experimental data (Joachimowicz and Pichot, 1992). This removes the suspicion evoked by using a similar method (method of moments) for the forward problem and the inverse method in the numerical simulations presented here.

REFERENCES

S. Caorsi, G.L. Gagnani and M. Pastorino, 1988. Electromagnetic Vision-Oriented Numerical Solution to Three-Dimensional Inverse Scattering, Radio Science, 6, 1094-1106

Caorsi S., Gagnani G.L. and Pastorino M., 1990. Two-Dimensional Microwave Imaging by a Numerical Inverse Scattering Solution, IEEE Trans. Microwave Theory Tech., 8, 981-989.

W.C. Chew, and Y.M. Wang, 1990. Reconstruction of Two-Dimensional Permittivity Distribution Using the Distorted Born Iterative Method, IEEE Trans. Med. Imaging, 2, 218-225.

A.N. Datta and B. Bandyopadhyay, 1986. Nonlinear Extension to a Moment Method Iterative Reconstruction Algorithm for Microwave Tomography, Proc. IEEE, 4, 604, 606.

A. Franchois and C. Pichot, 1992. Generalized Cross Validation Applied to a Newtontype Algorithm for Microwave Tomography, Proc. SPIE inverse Problems in Scattering and Imaging, San Diego, to appear.

D.K. Ghodgaonkar, O.P. Gandhi and M.J. Hagmann, 1983. Estimation of Complex Permittivities of Three-Dimensional Inhomogeneous Biological Bodies, IEEE Trans. Microwave Theory Tech., 6, 442-446

T.C. Guo and W.W. Guo, 1987. Physics of Image Information by Microwave Scattering. Medical Imaging, SPIE Proc., 4, 1-10.

T.M. Habashi and D.G. Dudley, 1989. Simultaneous Inversion of Permittivity and Conductivity Profiles Using a Renormalized Source-Yype Integral Equation Approach, Proc. URSI Int. Symp., Stockkholm, 578-580.

M.J. Hagmann, O.P. Gandhi and D.K. Ghodgaonkar, 1981. Application of Moment-Methods to Electromagnetic Imaging, IEEE MTT-S Symp. Digest, Washington, 482

N. Joachimowicz, Ch. Pichot, J.P. Hugonin, 1991. Inverse Scattering: an Iterative Numerical Method for Electromagnetic Imaging, IEEE Trans. Antennas Propagat., 12, 302-310

N. Joachimowicz and Ch. Pichot, 1992. Inverse Scattering Solution for Inhomogeneous Bodies, IEEE Ap-S/URSI Symposium Digest, Chicago, 1720.

S.A. Johnson and M.L. Tracy, 1983. Inverse Scattering Solutions by a Sinc Basis Moment Method, Part I, ultrason. Imaging, 5, 361-375, Part II, Ultrason. Imaging, 5, 376-392 ; 1984, Part III, Ultrason. Imaging, 6, 103-116

M.M. Ney, A.M. Smith and S.S. Stuchly, 1984. A Solution of Electromagnetic Imaging Ysing pseudo-inverse transformation, IEEE Trans. Med. Imaging., 4, 155-162.

D.G.H. Tan, R.D. Murch and R.H.T. Bates, 1989. Inverse Scattering for Penetrable Obstacles, URSI Int. Symp. Proc. Sueden, 160-162.

A.N. Tikhonov and V.Y. Arsenine, 1977. Solutions of Ill-Posed Problems (Washington: Winston-Wiley)

Y.M. Wang and W.C. Chew, 1989. An Iterative Solution of Two-Dimensional Electromagnetic Inverse Scattering Problem, Int. J. Imag. Syst. Tech., 1, 100-108.