Comparison of Optimization Procedures for 2x2 Sinclair, 2x2 Graves, 3x3 Covariance, and 4x4 Mueller (Symmetric) Matrices in Coherent Radar Polarimetry and its Application to Target Versus Background Discrimination in Microwave Remote Sensing and Imaging

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ABSTRACT

Basic principles of radar polarimetry are introduced and various optimization procedures for the propagation (scattering) range operator equation and the received power expressions are presented and compared. It is assumed that the radar is a complete coherent dual orthogonal (A,B) transmit/receive antenna system of high channel isolation and antenna side-lobe reduction, where in the case of wave interaction with a discrete stationary point target the propagation (scattering) matrix is given by the 2x2 coherent Jones (Sinclair) matrix [S(A,B)], the 2x2 complex Graves power [G(AB)], the 3x3 or 4x4 complex covariance matrix [Σ(A,B)], and the 4x4 real Mueller (Kennaugh) power density matrix [M] for the symmetric (monostatic reciprocal: $S_{AB} = S_{BA}$) or the asymmetric (general bistatic, monostatic non-reciprocal: S_{AB} c S_{BA}) cases, respectively. Four separate optimization procedures are here introduced for the symmetric case, demonstrating that for the coherent (deterministic) scattering scenario the solutions obtained from optimizing the pertinent power density expressions associated with either [S(A,B)], [G(AB)], [Σ(A,B)] and [M] are identical, and so approximately also for the partially polarized case. Pertinent contrast enhancement optimization procedures for discrimination between two classes of targets, the 'optimal polarimetric contrast enhancement coefficients': 'opcec' are introduced and expressed in terms of power density expression for the four scattering matrices [S(AB)], [G(AB)], [Σ(AB)] and [M] valid for the coherent and partially polarized cases. Whereas, for the partially coherent case more elaborate optimization procedures for the 3x3 covariance and/or 4x4 Mueller matrices need to be employed utilizing special properties of Lie group SU(n = 2,3,4) expansions, i.e., the 2x2 Pauli spin [σ_i; i=0,1,2,3], the 3x3 Hausdorff (Gell-Mann) [δ_i; i = 1,2,9] and the 4x4 Dirac [θ_i; i = 0,1,2,15] matrices.

Based on this complete description of isolated and distributed scatterers, target classification, target-versus-clutter discrimination, and optimal contrast enhancement algorithms are derived and shown to be of great utility in the proper interpretation of POL-RAD/SAR microwave signatures in terrestrial and planetary remote sensing.

INTRODUCTION

The basic principle of radar polarimetry is based on the concept of characteristic polarization states first introduced by Kennaugh (Kennaugh, 1981-1992), who demonstrated that there exist radar polarization states for which the radar receives minimum/maximum power. This min/max polarization state theory was extended primarily by Huynen (Huynen, 1978; Huynen, McNulty, Hanson, 1975), who introduced the "polarization fork" concept, and more recently by us (Boerner, 1980-81; Boerner, Liu, Zhang, 1992) and at DLR-Oberpfaffenhofen (Tragl, 1992; Gneburg, Ziegler, Tragl, Schroth, 1991). With the advent of dual polarization coherent radar (Giuli, 1986) and POL-RAD/SAR (van Zyl, 1986; Zebker, van Zyl, 1991) systems, radar polarimetry has become a subject of recurring and globally intensifying interest in recent years (Berner, 1985-1992). In spite of extensive studies of this theory, a final rigorous and complete formulation still is warranted (Boerner, Yan, Xi, Yamaguchi, 1992). Different approaches were introduced for determining these characteristic polarization states by using the voltage equation [1], the eigenvalue problem of the power scattering matrix (Huynen, 1965-78-75-92-90; Davidovitz, Boerner, 1986-83; Kostinski, Boerner, 1395-1404-1987, Mieras, 1470-

1471), the basis transformation techniques (Boerner, 1981-1980; Agrawal, Boerner, 1989-1992; Boerner, Xi, 1990-1992), the Mueller matrix approach for the "degenerate coherent Stokes vector case" (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan Xi, Yamaguchi, 1992), and more recently, the properly corrected polarimetric covariance matrix optimization procedure (Boerner, Liu, Zhang, 1992; Tragl, 1992-1990, Tragl, Gneburg, Schroth, Ziegler, 1991). All of these methods are compared and it is demonstrated how each of them contributes partially towards a complete understanding of coherent scattering matrix properties.

It is shown that there exist in total five unique pairs of characteristic polarization states for the symmetric coherent scattering matrix [S(A,B)] of which two pairs, corresponding to the cross-polarization (x-pol) null and co-polarization (co-99pol) maxima, are identical; whereas the x-pol max and x-pol saddle point pairs are distinct (Boerner, Xi, 1990-92). These three pairs of orthogonal characteristic polarization states are also mutually at right angles to one another on the polarization sphere. The fifth pair, the (in general) non-orthogonal co-pol null pair, lies in the plane spanned by the co-pol max, or equivalently the x-pol null, and the x-pol max pairs which determine the 'target characteristic plane (circle) of Kennaugh'(Kennaugh, 1992-81; Boerner, Xi, 1990-92; Boerner, Yan, Xi, Yamaguchi, 1992) and the angle between the co-polar nulls is bisected by the line joining the two co-pol maxs; and together with the orthogonal x-pol saddlepoint pair, being at right angles to this plane, they re-establish Huynen's 'polarization fork' concept (Huynen, 1965-1978, Huynen, McNulty, Hanson, 1975, Huynen, 1992-90; Boerner, Xi, 1990-92; Boerner, Yan, Xi, Yamaguchi, 1992). The distinctly different optimization approaches are compared by one illustrative example in which, besides the 'polarization forks', also the co-pol and x-pol power density plots (Agrawal, Boerner, 1989-90) and the relative co/cross-polarization phase (polarimetric correlation coefficient) plots (Agrawal, Boerner, 1989-90; Boerner, Yan, Xi, Yamaguchi, 1992) are presented.

More approaches still may be required to completely resolve all unanswered questions even for the coherent case, for example, such as those recently presented by McCormick in [18] for applications to radar meteorology; and so also a more rigorous group-theoretic approach of optimizing the Sinclair, covariance and Mueller matrices expanded in terms of Lie (SU(2),SU(3),SU(4/2)) groups associated with the Pauli spin matrices as pursued vigorously by Cloude (Cloude, 1986-90-88).

Next to determining the eigenvalue and optimization problems for the isolated matrices [S],[G],[Σ] and [M] - equally important - the exact and correct expressions for the enhancement of the 'optimal contrast between two classes of scatterers or scatterer ensembles must be determined as was first considered by Soviet radar polarimetrists (see (Boerner, 1992), and still requires extensive investigations for completion for either the coherent, partially polarized or partially coherent cases. Whereas, a unique optimization method for the general partially coherent case still does not exist, either for the matrices or the associated contrast enhancement coefficients, considerable progress was made in determining an optimization approach for the partially polarized case (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; (Boerner, 1985; Boerner, Yan, Xi, Yamaguchi, 1992; Kostinski, James, Boerner, 1988) for which it is assumed that the wave incident on a stochastic scatterer is completely polarized. Also, it is shown in (Boerner, Liu, Zhang, 1992; Cloude, 1986-92-90-91-88) that there exist 'physical realizability' conditions to which the elements of the 4x4 Mueller matrices are subjected in order to identify erroneous measurement results such as of the degree of polarization of the scattered wave to be greater than unity (Cloude, 1986-92-90-91-88), etc. These and similar physical realizability (Fry, Kattawar, 1981; Hovenier, van de Hulst, C.V.M., 1986) conditions apply, in general, also in the partially polarized case requiring a four-dimensional polarization sphere treatment (Czy, 1992-91; Zhivotovskiy, 1992-88-89) together with a SU(4/2) group-theoretic treatment (Cloude, 1986-92-90-91-88) which will be considered in another paper (Boerner, Liu, Zhang, 1992).

This paper concludes by identifying useful applications of these basic principles of radar polarimetry to practical problems in ultrawideband polarimetric impulse radar target imaging (Boerner, Liu, Zhang, Naik, 1992); to high resolution air/space-borne POL-SAR imaging (Boerner, 1987; Walther, Segal, Boerner, 1992); and in polarimetric matched filtering (Kostinski, James, Boerner, 1988; Walther, Segal, Boerner, 1992).

1. FORMULATION OF THE SCATTERING MATRICES

A plane electromagnetic wave (H,V) can be expressed (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992) in the orthogonal polarization basis (H,V) as (Fig.1)

$$\vec{E}(HV) = A\,(\hat{h}_H + \rho_{HV}\,\hat{h}_V)\,.\,h\ patio \tag{1a}$$

The complex polarization transformation ratio ρ_{HV} is given by

$$\rho_{HV} = |\rho_{HV}|\,exp j\delta_{HV} = |E_v/E_H|\,exp j\,(\delta_v - \delta_H) = \\ = \tan\alpha_{HV}\,exp j\delta_{HV} = (\tan\phi + j tan\tau)/(1 - j tan\phi\tan\tau) \tag{1b}$$

where α_{HV} and δ_{HV} can be expressed in terms of the tilt and ellipticity angles (ϕ, τ) of Fig. 1 as $\cos 2\alpha_{HV} = \cos 2\phi \cos 2\tau$ and $\tan\delta_{HV} = \tan 2\tau / \sin 2\phi$ so that the orthogonality condition in any orthogonal polarization basis (A,B; B $= A$) satisfies the following orthogonality condition (Boerner, Xi, 1990; Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992)

$$\rho_{AB} \cdot \rho_{AB}^{\perp *} = -1 \ , \ \textit{with} \ \rho_{AB}^{\perp} = -\frac{1}{\rho^*} \ . \qquad (1c)$$

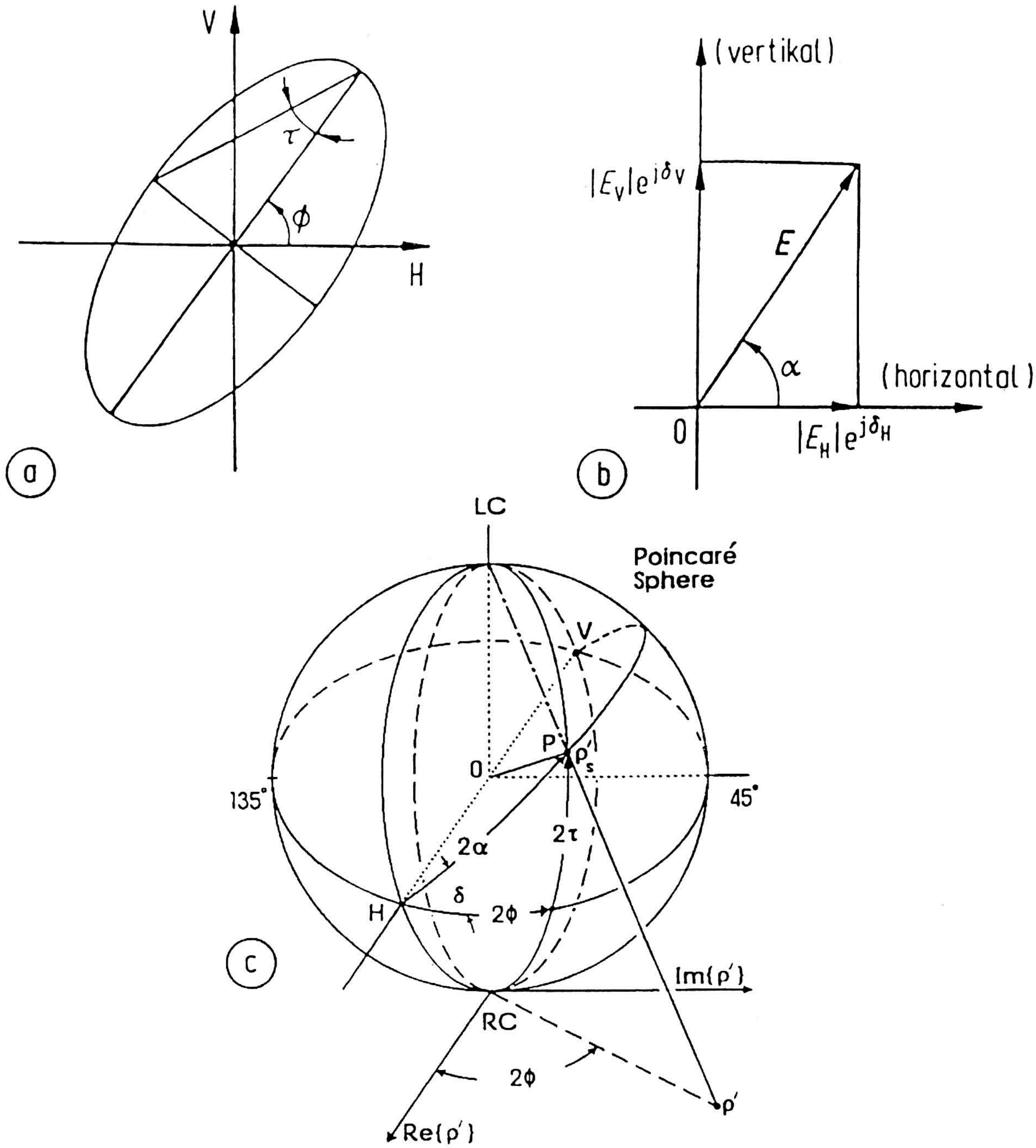


Fig. 1 - POLARIZATION STATE DESCRIPTORS: (a) Parametric presentation of the polarization ellipse ; (b) Representation of the polarization ratio in the horizontal-vertical (H-V) basis ; (c) Representation of a polarization state on the Poincaré sphere with correspondence of point on the complex polar plane, ρ′, with point P (ρ'_s) on the polarization sphere.

1.1 Sinclair (Jones) Matrix [S(HV)] and the Graves Power Matrix [G(HV)]

The scattering matrix [S(HV)], normalized with respect to range and antenna gain functions [33-37], can then be expressed in terms of the incident ($E_i(HV)$) and scattered ($E_s(HV)$) fields [8,17] by

$$\vec{E_s}(HV) = [\,S\,(HV)\,]\,\vec{E_i}(HV)\,,\ [\,S\,(HV)\,] = \begin{bmatrix} S_{HH}\,S_{HV} \\ S_{VH}\,S_{VV} \end{bmatrix} \tag{2a}$$

where for the reciprocal monostatic (symmetric matrix) case $S_{HV} = S_{VH}$, which is being considered here only. The power received at the antenna terminal is then given (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992) by

$$P = |\,V_R\,|^2 = |\,\vec{e}_R^{\,T}\,[\,S\,]\,\vec{E_T}\,|^2 \tag{2b}$$

with the terminal voltage [5] being expressed in terms of the antenna height $\vec{h}$ [5,8] as

$$V_R = \vec{h}_R^{\,T}\,\vec{E_s} = \vec{h}_R^{\,T}\,[\,S\,]\,\vec{E_T} = \vec{e}_R^{\,T}\,[\,S\,]\,\vec{E_T}\,;\ \vec{e_R} = \frac{\vec{E_R}}{\|\,\vec{E_R}\,\|} \tag{2c}$$

Kennaugh (Kennaugh, 1992) first showed that the scattering matrix [S], in general, is not symmetric and also not Hermitian; thus, in order to determine the proper set of real eigenvalues he first proposed to introduce a coherent complex power density matrix. As is discussed in detail in Chan (Chan, 1981), Graves (Graves, 1956) later on re-established Kennaugh's finding and introduced the normalized energy density expression W(ρ) as:

$$W(\rho) = \vec{E}_s^{\,+}\,\vec{E_s} = (\,[\,S\,]\,\vec{E_T}\,)^+\,(\,[\,S\,]\,\vec{E_T}\,) = \\ = \vec{E}_T^{\,+}\,(\,[\,S\,]^+\,[\,S\,]\,)\,\vec{E_T} = \vec{E}_T^{\,+}\,[\,G\,]\,\vec{E_T} \tag{2d}$$

and the complex power density matrix was defined by $[\,G\,] = [\,S\,]^+\,[\,S\,]$. The Graves matrix [G] was then used in (Kostinski, Boerner, 1986-87, Mieras) to develop the 'three-stage optimization procedure', summarized later on.

1.2 Unitary Basis Transformation and Invariants

The 2x2 unitary basis transformation matrix [U] for transforming from (HV) to (AB) becomes (Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992)

$$\vec{E}(HV) = [\,U\,]\,\vec{E}\,'(AB) \tag{3a}$$

with

$$[\,U\,] = \frac{1}{\sqrt{1+\rho\rho^*}} \begin{bmatrix} e^{j\psi_1} & \rho\, e^{j\psi_1} \\ -\rho^*\, e^{j\psi_4} & e^{j\psi_4} \end{bmatrix} \tag{3b}$$

so that

$$[\,S'\,)\,(AB)\,] = [\,U\,]^T\,[\,S\,(HV)\,]\,[\,U\,] = \begin{bmatrix} S'_{AA}\,S'_{AB} \\ S'_{AB}\,S'_{BB} \end{bmatrix}, \tag{3c}$$

satisfying the following transformation invariants (Kostinski, Boerner, 1986-87, Mieras; Boerner, Xi, 1990-92; Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992)

$$Span\,[\,S'(AB=BA)\,] = |\,S'_{AA}\,|^2 + 2\,|\,S'_{AB}\,|^2 + |S'_{BB}\,|^2 \\ = |\,S_{HH}\,|^2 + 2\,|\,S_{HV}\,|^2 + |\,S_{VV}\,|^2 \tag{3d}$$

$$|\,Det\,[\,S'\,(AB=BA)\,]\,| = |\,S'_{AA}\,S'_{BB} - (S'_{AB})^2\,| = \\ = |\,S_{HH}\,S_{VV} - (S_{HV})^2\,|\,. \tag{3e}$$

1.3 Covariance Feature Vector and Covariance Matrix for Symmetric Case

Utilizing these invariants (3d) and (3e), first established applied in radar polarimetry in [3d], the concept of the polarimetric feature vector $\vec{\Omega}$ and corresponding polarimetric covariance matrix $[\,\Sigma\,]$ may be introduced (Boerner, Liu, Zhang, 1992; Tragl, 1992). At each instantaneous state of time any stochastic target is completely described by a corresponding scattering matrix [S'(AB)] or equivalently by a polarimetric feature vector $\vec{\Omega}(AB)$. For the symmetric matrix case (AB = BA), a three-dimensional covariance feature vector (Boerner, Liu, Zhang, 1992; Tragl, 1992) is introduced

$$\vec{\Omega}(AB=AB) = (S'_{AA}\ \sqrt{2}\ S'_{AB}\ \ S'_{BB})^T\,, \tag{4a}$$

satisfying the energy conservation (power density) invariance under a unitary basis transformation

$$\|\,\vec{\Omega}\,\|^2 = Span\,[\,S\,(AB=BA)\,] = |\,S'_{AA}\,|^2 + 2\,|\,S'_{AB}\,|^2 + |\,S'_{BB}\,|^2\,, \tag{4b}$$

Note that, in the literature formulations neglecting the multiplicative factor $\sqrt{2}$ exist (Swartz, Yueh, Kong, Novak, Shin, 1988; Novak, Sechtin, Cardullo, 1987, Novak, Burl, Chaney, Owirka, 1990, Novak 1992) which are erroneous, because those formulations violate fundamental energy and minimum phase conservation principles, as was repeatedly stated by the author, and this important point was reinforced recently also by Cloude (Cloude, 1986). The corresponding, correctly defined polarimetric covariance matrices, [Σ(AB)] and [Σ(HV)], for the symmetric case, in the (AB) and (HV) bases, respectively, are then defined for the instantaneous state (Boerner, Liu, Zhang, 1992; Tragl, 1992), respectively by

$$[\Sigma(AB)] = \vec{\Omega}(AB)\,\vec{\Omega}(AB)^{+} = \begin{bmatrix} |S'_{AA}|^2 & \sqrt{2}\,S'_{AA}S'^{*}_{AB} & S'_{AA}S'^{*}_{BB} \\ \sqrt{2}\,S'_{AB}S'^{*}_{AA} & 2|S'_{AB}|^2 & \sqrt{2}\,S'_{AB}S'^{*}_{BB} \\ S'_{BB}S'^{*}_{AA} & \sqrt{2}\,S'_{BB}S'^{*}_{AB} & |S'_{BB}|^2 \end{bmatrix} \tag{4c}$$

and similarly we can define

$$[\Sigma(HV)] = \vec{\Omega}(HV)\,\vec{\Omega}(HV)^{+} = \begin{bmatrix} |S_{HH}|^2 & \sqrt{2}\,S_{HH}S^{*}_{HV} & S_{HH}S^{*}_{VV} \\ \sqrt{2}\,S_{HV}S^{*}_{HH} & 2|S_{HV}|^2 & \sqrt{2}\,S_{HV}S^{*}_{VV} \\ S_{VV}S^{*}_{HH} & \sqrt{2}\,S_{VV}S^{*}_{HV} & |S_{VV}|^2 \end{bmatrix} \tag{4d}$$

satisfying the transformation (with $^{+}$ denoting the Hermitian conjugate)

$$\vec{\Omega}(AB) = [\Upsilon]\,\vec{\Omega}(HV), \quad [\Sigma(AB)] = [\Upsilon][\Sigma(HV)][\Upsilon]^{+} \tag{4e}$$

with $[\Upsilon(\rho)][\Upsilon(\rho)]^{+} = [I]$ and $|Det\{[\Upsilon(\rho)]\}| = 1$ and the explicit expression is given as (Boerner, Liu, Zhang, 1992; Tragl, 1990; Tragl, LGneburg, Schroth, Ziegler; LGneburg, Ziegler, Tragl, Schroth, 1991).

$$[\Upsilon(\rho)] = \frac{1}{(1+\rho\rho^{*})} \begin{bmatrix} e^{2j\psi_1} & \sqrt{2}\,\rho\, e^{2j\psi_1} & \rho^2 e^{2j\psi_1} \\ -\sqrt{2}\,\rho^{*} e^{j(\psi_1+\psi_4)} & (1-\rho\rho^{*})\, e^{j(\psi_1+\psi_4)} & \sqrt{2}\,\rho\, e^{j(\psi_1+\psi_4)} \\ \rho^{*2} e^{2j\psi_4} & -\sqrt{2}\,\rho^{*} e^{2j\psi_4} & e^{2j\psi_4} \end{bmatrix} \tag{4f}$$

1.4 The Stokes Vector

According to (Boerner, Yan, Xi, Yamaguchi, 1992), the Stokes vector $\vec{g}$ for a partially polarized wave may be defined as the sum of the completely polarized ($\vec{g}_q$) and the unpolarized ($\vec{g}_u$) components expressed in terms of the coherency matrix [J(HV)] or the coherency vector $\vec{J}(HV)$ (Kostinski, James, Boerner, 1988), (Azzam, Basharra, 1977; Ishimaru, 1991)

$$[J] = [\langle \vec{E}\,\vec{E}^{*}\rangle] = \begin{bmatrix} \langle E_H E_H^{*}\rangle & \langle E_H E_V^{*}\rangle \\ \langle E_V E_H^{*}\rangle & \langle E_V E_V^{*}\rangle \end{bmatrix} = \begin{bmatrix} J_{HH} & J_{HV} \\ J_{VH} & J_{VV} \end{bmatrix};$$

$$\vec{J}^{T} = [J_{HH}\,J_{HV}\,J_{VH}\,J_{VV}], \quad \langle \ldots \rangle = \lim_{T=0}\left(\frac{1}{2T}\int_{-T}^{T}(\ldots)\,dt\right) \tag{5a}$$

$$\vec{g} = \vec{g}_q + \vec{g}_u = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} q g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} + \begin{bmatrix} (1-q)\,g_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \tag{5b}$$

$$\begin{bmatrix} J_{HH}+J_{VV} \\ J_{HH}-J_{VV} \\ J_{HV}+J_{VH} \\ jJ_{HV}-jJ_{VH} \end{bmatrix} = [A]\,\vec{J},\; [A] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix}$$

The degree of polarization q, later on required for deriving a more useful expression in terms of the corrected covariance matrix expressions, is given by

$$q = \frac{\sqrt{g_1^2+g_2^2+g_3^2}}{g_0} = \sqrt{1-\frac{4\det[J]}{(J_{HH}+J_{VV})^2}}, \quad 0 \le q \le 1,\; J_{HV} = J^{*}_{VH} \tag{5c}$$

and the complex degree of coherency μ by

$$\mu = |\mu|\,e^{j\beta} = \frac{J_{HV}}{\sqrt{J_{HH}J_{VV}}},\; 0 \le |\mu| \le q \le 1, \quad \frac{(1-q^2)}{(1-|\mu|^2)} = \frac{4J_{HH}J_{VV}}{(J_{HH}+J_{VV})^2}\,.. \tag{5d}$$

For the coherent case $|\mu| = 1$ and $q = 1$, with $g_0^2 = g_1^2 + g_2^2 + g_3^2$ expressed in the $1_{HV}(\alpha_{HV}, \delta_{HV})$ formulation, presenting a fixed point on the polarization sphere, (Fig. 1), the Stokes vector becomes

$$\vec{g}(q=1) = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} |E_H|^2 + |E_V|^2 \\ |E_H|^2 - |E_V|^2 \\ 2|E_H||E_V|\cos\phi \\ 2|E_H||E_V|\sin\phi \end{bmatrix} = A^2 \begin{bmatrix} 1 \\ \cos 2\tau \cos 2\phi \\ \cos 2\tau \sin 2\phi \\ \sin 2\tau \end{bmatrix} = A^2 \begin{bmatrix} 1 \\ \cos(2\alpha_{HV}) \\ \sin(2\alpha_{HV})\cos(\delta_{HV}) \\ \sin(2\alpha_{HV})\sin(\delta_{HV}) \end{bmatrix} \tag{5e}$$

1.5 The Mueller Matrices [M], [Mc], [Mx], and [Mm]

For the partially coherent case the scattered ($\vec{g}_s$) and the incident ($\vec{g}_i$) fields for forward (backward) scattering are related by the real Mueller (Kennaugh) matrix [M], which for the 'degenerate coherent case' (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992) can be expressed in terms of the scattering matrix [S] by

$$\vec{g}_s = [M]\,\vec{g}_i,\; [M] = [A(AB)]\,([S(AB)] \otimes [S(AB)]^{*})\,[A(AB)]^{-1}, \tag{6a}$$

where ⊗ denotes (tensorial) Kronecker matrix multiplication and the Kronecker expansion matrix [A(AB)] differs for different bases (AB) (Boerner, Yan, Xi, 1992); and for the (HV) basis it is given by (5a).

The corresponding received power expression is then given (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992) by

$$P = \frac{1}{2} \vec{g}^{T} [\overline{M}_P] \vec{g} \qquad (6b)$$

which, as was shown in (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992), can be reexpressed for the co-polarized (co-pol: **c**) and cross-polarized (cross-pol: **x**) channels

as

$$P_C = |\vec{h}_T^T [S] \vec{E}_T|^2 = \frac{1}{2} \vec{g}_T^T [\overline{M}_C] \vec{g}_T, \qquad (6c)$$

with

$$[\overline{M}_C] = ([A]^{-1})^T ([S] \otimes [S]^*) [A]^{-1} = [C][M]$$

and

$$P_X = |\vec{h}_{T_\perp}^T [S] \vec{E}_T|^2 = \frac{1}{2} \vec{g}_T^T [\overline{M}_X] \vec{g}_T \qquad (6d)$$

with

$$[\overline{M}_X] = ([A]^{-1})^T \left[\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} ([S] \otimes [S]^*) \right]$$

$$[A]^{-1} = [X][M],$$

where

$$[C] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad [X] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In case the receiver antenna polarization state $\vec{h}_R$ is matched to the incoming scattered wave, then

$$\vec{h}_R = \vec{E}_S^* / \| \vec{E}_S \| \quad , \quad P_m = \frac{1}{2} \vec{g}_T^T [\overline{M}_m] \vec{g}_T \qquad (6e)$$

with

$$[\overline{M}_m] = \frac{1}{2} \left[[\overline{M}_C] + [\overline{M}_X] \right] = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} [M] \qquad (6f)$$

which completes the introduction of the pertinent scattering matrices (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992) used in radar polarimetry.

1.6 Partially polarized case

For the partially coherent and also for the partially polarized cases, the following optimization criteria result for the scattered energy density arriving at the receiver according to (5a), which may described in terms of four categories [25,8,17]:

gso Total energy density in the scattered wave before it reaches the receiver;(7a)

qgso Completely polarized part of the intensity; i.e., the adjustable intensity because one may adjust the polarization state of the receiver to ensure polarization matching;(7b)

(1-q)gso Noise of the unpolarized part: regardless of the receiver polarization state, one half of the unpolarized part, i.e., gso(1-q)/2 is always accepted.(7c)

(1+q)gso/2 Maximum of the total receptable intensity {qgso} + {(1-q)gso/2} = {(1+q)gso/2}, i.e., the sum of the matched polarized part is mismatched (canceled with proper receiver tuning), the total received power is minimal and equal half the unpolarized power, i.e. (1-q)gso/2.(7d)

We note, here that Cloude established another set of the optimization criteria based on the target matrix decomposition (Cloude, 1988) which is assessed later on as the two methods must provide identical results if valid.

2. THE PROPERTIES OF THE NORMALIZED COVARIANCE MATRIX EXPRESSION IN TERMS OF THE CO/CROSS-POLAR POWER DENSITY AND OF THE RELATIVE PHASE CORRELATION COEFFICIENTS

2.1 The Polarimetric Covariance Matrix for the Stochastic Case

The recent availability of advanced coherent dual polarization radar systems, allowing the decomposition of the received wave into two orthogonal complex components (co-polar and cross-polar transceiver channels), and highly simplified polarimetric ensemble averaging for $< S_{ij} S_{ke}^* >$, facilitates the introduction of the polarimetric feature vector $\vec{\Omega}$ of (12b) for interpreting the ensemble (time)-averaged scattering behavior of reciprocal random targets ($S'_{AB} = S'_{BA}$). Utilizing the scattering matrix invariances of (3d/e), the polarimetric feature vector $\vec{\Omega}$ was introduced subject to the normality condition (4b), permitting the formulation of the covariance matrix [Σ] according to (5a/b) also for the stochastic symmetric case with

< ... > denoting either appropriate ensemble or time averaging (see eq.(5b)) of a stochastic variable (Boerner, Liu, Zhang, 1992; Tragl, 1992)

$$[\,\Sigma\,](AB)\,] = \\ = \begin{bmatrix} <|S'_{AA}|^2> & \sqrt{2}<S'_{AA}S'^{*}_{AB}> & <S'_{AA}S'^{*}_{BB}> \\ \sqrt{2}<S'_{AB}S'^{*}_{AA}> & 2<|S'_{AB}|^2> & \sqrt{2}<S'_{AB}S'^{*}_{BB}> \\ <S'_{BB}S'^{*}_{AA}> & \sqrt{2}<S'_{BB}S'^{*}_{AB}> & <|S'_{BB}|^2> \end{bmatrix} \tag{8a}$$

and

$$[\,\Sigma\,](HV)\,] = \\ = \begin{bmatrix} <|S_{HH}|^2> & \sqrt{2}<S_{HH}S^{*}_{HV}> & <S_{HH}S^{*}_{VV}> \\ \sqrt{2}<S_{HV}S^{*}_{HH}> & 2<|S_{HV}|^2> & \sqrt{2}<S_{HV}S^{*}_{VV}> \\ <S_{VV}S^{*}_{HH}> & \sqrt{2}<S_{VV}S^{*}_{HV}> & <|S_{VV}|^2> \end{bmatrix} \tag{8b}$$

These polarimetric covariance matrices are directly related to the statistical properties of the scattering matrix elements and its formulation is consistent with the Stokes reflection matrix decomposition into its co-polar matrix $[\,M_C\,]$ and cross-polar matrix $[\,M_X\,]$ as introduced in (Yan, Boerner, 1991; Boerner, Yan, Xi, 1992) and defined in (6d). Following the approach of (Agrawal, Boerner, 1989-92) and (Tragl, 1992-90, Tragl, Lcneburg, Schroth, Ziegler, 1991) of utilizing the reduced transformation matrix (Davidovitz, Boerner, 1986-83, Agrawal, Boerner, 1989-92; Tragl, 1992-90, Tragl, Lcneburg, Schroth, Ziegler, 1991) with $\psi_1 = 0$ and $\psi_4 = 0$, for the linear (HV) basis, eq.(4e) may be reformulated as

$$\vec{\Omega}(AB) = \vec{\Omega}(\rho) = [\,\Upsilon(\rho)\,]\,\vec{\Omega}(HV)\,, \tag{8c}$$

and

$$[\,\Sigma\,(AB)\,] = [\,\Sigma\,(\rho)\,] = <\vec{\Omega}(\rho)\,\vec{\Omega}^{+}(\rho)> = \\ = [\,\Upsilon(\rho)\,]<\vec{\Omega}(HV)\,\vec{\Omega}^{+}(HV)>[\,\Upsilon(\rho)\,]^{+} \tag{8d}$$

with

$$[\,\Upsilon(\rho)\,] = \frac{1}{(1+\rho\rho^{*})}\begin{bmatrix} 1 & \sqrt{2}\,\rho & \rho^2 \\ -\sqrt{2}\,\rho^{*} & (1-\rho\rho^{*}) & \sqrt{2}\,\rho \\ \rho^{*2} & -\sqrt{2}\,\rho^{*} & 1 \end{bmatrix}, \tag{8e}$$

where $[\,\Upsilon(\rho)\,]\,[\,\Upsilon(\rho)\,]^{+} = [\,I\,]$ and $Det\{[\,\Upsilon(\rho)\,]\} = 1$. With the introduction of the above matrices and power density expressions, the proper covariance matrix $[\,\Sigma\,(AB)\,]$ can be reexpressed (Boerner, Liu, Zhang, 1992) in terms of the co/cross-polar channel power expression, $P_c(\rho)$ and $P_x(\rho)$, for the case of transmitting polarization state A and receiving $B = A$; whereas for reversed (orthogonal) order of transmitting B and receiving $A = B$, the corresponding orthogonal expressions are denoted by $P_c(\rho)$, $P_x(\rho)$ and similarly the off-diagonal relative phase co/cross polar channel correlation expressions $R_c(\rho)$ and $R_x(\rho)$ become $R_c(\rho)$ and $R_x(\rho)$, where

$$[\,\Sigma\,(\rho)\,] = \begin{bmatrix} P_C(\rho) & \sqrt{2}\,R_X(\rho) & R_C(\rho) \\ \sqrt{2}\,R_X(\rho)^{*} & 2P_X(\rho) & \sqrt{2}\,R_X^{\perp}(\rho)^{*} \\ R_C(\rho)^{*} & \sqrt{2}\,R_X^{\perp}(\rho) & P_C^{\perp}(\rho) \end{bmatrix}$$

and since $\rho\,\rho^{*} = -1$, we find with equations (4c - 4f)

$$\left[\,\Sigma\left(\rho^{\perp} = -\frac{1}{\rho^{*}}\right)\right] = \\ = \begin{bmatrix} P_C^{\perp}(\rho) & -\frac{\rho}{\rho^{*}}\sqrt{2}\,R_X^{\perp}(\rho) & \frac{\rho^2}{\rho^{*2}}R_C(\rho)^{*} \\ -\frac{\rho^{*}}{\rho}\sqrt{2}\,R_X^{\perp}(\rho)^{*} & 2P_X(\rho) & -\frac{\rho}{\rho^{*}}\sqrt{2}\,R_X(\rho)^{*} \\ \frac{\rho^{*2}}{\rho^2}R_C(\rho)^{*} & -\frac{\rho^{*}}{\rho}\sqrt{2}\,R_X(\rho) & P_C(\rho) \end{bmatrix}, \tag{9b}$$

satisfying the following orthogonality relations

$$P_C(-1/\rho^{*}) = P_C^{\perp}(\rho)\,, \quad |R_X(-1/\rho^{*})| = |R_X^{\perp}(\rho)| \tag{9c}$$

and symmetry relations

$$P_X(-1/\rho^{*}) = P_X(\rho)\,, \quad |R_C(-1/\rho)| = |R_C(\rho)|\,, \tag{9d}$$

so that the stochasticity coefficients, defined in (5c) and (5d), may be reformulated in terms of the normalized covariance power density expression later on required for determining 'optimal contrast polarimetric enhancement coefficients' (*opcec*),

$$\mu_{AB}(\rho) = \frac{R_X(\rho)}{\sqrt{P_C(\rho)\,P_X(\rho)}} \tag{9e}$$

and

$$q_{AB}(\rho) = \frac{\sqrt{(P_C(\rho)-P_X(\rho))^2 + 4\,|R_X(\rho)|^2}}{(P_C(\rho)+P_X(\rho))} \tag{9f}$$

$$0 \le |\,\mu_{AB}(\rho)\,| \le q_{AB}(\rho) \le 1\,. \tag{9g}$$

2.2 Eigenvalues and Eigenvectors of the Polarimetric Covariance Matrix

The (corrected) polarimetric covariance matrix $[\,\Sigma\,]$ is Hermitian and positive semi-definite and thus possesses three real, non-negative eigenvalues $0 \le \nu_1 \le \nu_2 \le \nu_3$ corresponding to a given matrix $[\,\Sigma\,]$ or equivalently [M], i.e., $\nu_i([\,\Sigma\,], i = 1,2,3)$, where it can be shown (Boerner, Liu, Zhang, 1992; Tragl, 1992-90, Tragl, LGeneburg, Schroth, Ziegler, 1991) that

$$0 \le \nu_1 \le \min_{\rho} P_C(\rho) \le P_C(HV) \le \max_{\rho} P_C(\rho) \le \nu_3 \\ \le \|\,\vec{\Omega}(HV)\,\|^2\,; \tag{10a}$$

and similar inequalities hold for P_c and P_x. A succinct interpretation of the target invariant eigenvalues $\nu_i(1,2,3)$ of the covariance matrix of random target polarimetric backscattering features is given in (Boerner, Liu, Zhang, 1992; Tragl, 1992-90, Tragl, LGeneburg, Schroth, Ziegler, 1991), showing that the smallest eigenvalue ν_1 indicates the degree of randomness (Boerner, Liu, Zhang, 1992).

For a deterministic target, with the covariance matrix defined by (4d) as $[\Sigma(HV)] = \vec{\Omega}(HV)\vec{\Omega}^{+}(HV)$, one obtains by involving a spectral theorem of matrix algebra (Horn, Johnson, 1985) that $\nu_1 = \nu_2 = 0$ and $\nu_3 = \|\vec{\Omega}(HV)\|^2$ for which true null polarization states $\rho_{cn1,2}$ exist (Tragl, 1992-90, Tragl, LGneburg, Schroth, Ziegler, 1991). The eigenvalue difference $\Delta\nu = (\nu_{max} - \nu_{min}) = (\nu_3 - \nu_1)$ of extremal covariance matrix eigenvalues determines the range in which the mean power return $P_c(\rho)$ and $2P_x(\rho)$ can be varied by polarimetric transceiver antenna adjustments also in consistency with the fundamental optimization criteria of (7a - 7d), where in particular (Boerner, 1981-80; Boerner, Yan, Xi, Yamaguchi, 1992; Boerner, Liu, Zhang, Naik, 1992; Walther, Segal, Boerner, 1992)

$$\begin{aligned} Trace[\Sigma(AB)] &= Trace[\Sigma(HV)] = Trace(<\vec{\Omega}\vec{\Omega}^{+}>) \\ &= <Trace(\vec{\Omega}\vec{\Omega}^{+})> = <\|\vec{\Omega}\|^2> = <Span[S]> \\ &= <|S'_{AA}|^2> + 2<|S'_{AB}|^2> + <|S'_{BB}|^2> \qquad (10b) \\ &= <|S_{HH}|^2> + 2<|S_{HV}|^2> + <|S_{VV}|^2> \\ &= \nu_1([\Sigma]) + \nu_2([\Sigma]) + \nu_3([\Sigma]) = invariant\ . \end{aligned}$$

In addition, the span of the covariance matrix $[\Sigma]$ is also an invariant (Boerner, Liu, Zhang, 1992; Tragl, 1992-90, Tragl, LGneberg, Schroth, Ziegler, 1991; Boerner, Walther, Segal, 1992), where

$$Span[\Sigma] = \sum_{i=1}^{3} \nu_i^2 = invariant\ , \qquad (10c)$$

and so is the ratio of the span versus the trace of the covariance matrix $[\Sigma]$ an invariant such that the '**covariance matrix invariance ratio (cmir)**' may be defined as

$$cmir = \frac{\sqrt{Span[\Sigma]}}{Trace[\Sigma]} = \frac{\sqrt{Span[\Sigma]}}{Span[S]} = \frac{\sqrt{\sum_{i=1}^{3}\nu_i^2}}{\sum_{i=1}^{3}\nu_i} = invariant \leq 1\ . \qquad (10d)$$

In POL-RAD/SAR signal and image processing '**cmir**' plays a significant role specifically as a measure (standard) for speckle reduction (Walther, Segal, Boerner, 1992; Swartz, Yueh, Kong, Novak, Shin, 1988). We note that similar expressions were derived by Novak et al. (Swartz, Yueh, Kong, Novak, Shin, 1988) utilizing a polarimetrically incorrect formulation based on decision theoretic approaches, as was also clearly identified recently by Cloude (Cloude, 1991-88). In addition, another set of powerful 'covarience matrix' and Mueller matrix realization conditions can be derived (Cloude, 1986-92-90-91-88; Hovenier, van de Hulst, C.V.M., 1986) and will play a key role in developing rigorous polarimetric radar calibration standards (Ulaby, Moore, Fung, 1986; Wiesbeck, Khny, 1991, Riegger, Wiesbeck, Khny, 1992).

3. OPTIMAL OR CHARACTERISTIC POLARIZATION STATES FOR THE COHERENT CASE

The Optimal Polarization State problem is to find polarization states transmitted and received, for a target of known scattering matrix [S] such that the voltage developed across the receiving antenna terminals is maximized (or minimized) (Boerner, Yan, Xi, Yamaguchi, 1992); or equivalently the power densities.

3.1 The Three-Step Optimization Approach for Graves Power Matrix [G]

This method enables one to treat the symmetric, asymmetric, monostatic and bistatic cases in an identical manner (Kostinski, Boerner, 1986-87, Mieras) but it is limited to the determination of the main polarization states for the matched antenna case only (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992).

Step 1

The total energy density W in the scattered wave is given according to (2d) by $\vec{E}_S^{+}\vec{E}_S$, where

$$W = \vec{E}_S^{+}\vec{E}_S = ([S]\vec{E}_T)^{+}[S]\vec{E}_T = \vec{E}_T^{+}[S]^{+}[S]\vec{E}_T = \vec{E}_T^{+}[G]\vec{E}_T \qquad (11a)$$

with the following eigenvalue problem:

$$[G]\vec{E}_{T,OPT} = \lambda\vec{E}_{T,OPT} \qquad (11b)$$

of solution

$$\lambda_{1,2} = \frac{1}{2}\left(Trace[G] \pm \sqrt{Trace^2[G] - 4\,Det[G]}\right)\ , \qquad (11c)$$

where

$$\lambda_1 + \lambda_2 = Trace[G] = Span[S] = |S'_{AA}|^2 + |S'_{AB}|^2 + |S'_{BA}|^2 + |S'_{BB}|^2 \qquad (11d)$$

$$= |S_{HH}|^2 + |S_{HV}|^2 + |S_{VH}|^2 + |S_{VV}|^2 = |S_{LL}|^2 + |S_{LR}|^2 + |S_{RL}|^2 + |S_{RR}| = invariant$$

$$\lambda_1 \lambda_2 = (Det[\,G\,] = (Det[\,S\,])\,(Det[\,S\,])^* = \tag{11e}$$

$$= (S'_{AA}\,S'_{BB} - S'_{AB}\,S'_{BA})\,(S'^*_{AA}\,S'^*_{BB} - S'^*_{AB}\,S'^*_{BA}) = invariant\ ,$$

from which "**Kennaugh's Polarimetric Excess:** σ_k" or the '**Effective Polarimetric Radar Cross-section (eprc)**' (Kennaugh, 1981), useful for geometrical power manipulations on the Poincaré) sphere (Boerner, 1992; Czy, 1992-91; Zhivotovskiy, 1992-89), can be defined as

$$\sigma_k = (Span[\,S\,] + 2\,|\,Det[\,S\,]\,|\,)\ . \tag{11f}$$

which was used extensively by *Czyz* in his alternate formulation of the fundamental polarimetric radar problem, also by Zhivotovsky (Zhivotovskiy, 1992-89), and in Wanielik's Lorentz transformation models [44] for the asymmetric bistatic scattering matrix cases (Davidovitz, Boerner, 1986-83; Cloude, 1986-92-90-91-88; Czy, 1992-91; Zhivotovskiy, 1992-89; Cho, 1990). In fact, the correct treatment of the asymmetric matrix optimization procedures require the rigorous introduction of 'Kennaugh's Polarimetric Excess' (Kennaugh, 1981) as was also shown in (Davidovitz, Boerner, 1983).

Step 2

Compute this scattered wave by using the known scattering matrix [S] and $\vec{E}_{T,OPT}$ from (11)

$$\vec{E}_{S,OPT} = [\,S\,]\,\vec{E}_{OPT} \tag{11g}$$

Step 3

$$\vec{h}_{R,OPT} = \frac{\vec{E}^*_{S,OPT}}{\|\vec{E}_{S,OPT}\|} = \frac{(\,[\,S\,]\,\vec{E}_{T,OPT}\,)^*}{\|\,[\,S\,]\,\vec{E}_{T,OPT}\,\|} \tag{11h}$$

This polarization match (11b) completes the three-step optimization process for the Graves power matrix approach (Boerner, Yan, Xi, Yamaguchi, 1992) and it provides exactly the same result as is obtained from the " matched degenerate Muller matrix" optimization of (6f).

3.2 The Critical Point or Basis Transformation Method for the Optimization of [S(AB)] Using the Generalized Transformation Matrix Formulation [6]

3.2.1 Generalized ρ - Transformation

With this method all existing characteristic states can be determined for which the radar receiver obtains maximum/minimum power backscattered from the targets and for which optimal polarization phase (δ) instabilities (cross-polar saddlepoint extrema) may occur (Fig.2a). In our case, the power expression (6b) can be written equivalently as

$$P = |\,V\,|^2 = |\,\vec{E}_R^{\,T}\,[\,S\,]\,\vec{e}_T\,|^2 = |\,\vec{E}'^{\,T}_R\,[\,S'\,]\,\vec{e}'_T\,|^2\ , \tag{12a}$$

where $'$ represents reference to any new basis (AB) which is obtained after unitary T - congruence transformation from the original basis (HV) (Kostinski, Boerner, 1986-87, Mieras)

$$[\,S'\,(AB)\,] = \begin{bmatrix} S'_{AA} & S'_{AB} \\ S'_{BA} & S'_{BB} \end{bmatrix} = [\,U\,]^T \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} [\,U\,]\ , \tag{12b}$$

where $S_{HV} = S_{VH}$ and $S'_{AB} = S'_{BA}$ for the monostatic scattering case (Yamaguchi, Sasagawa, Sengoku, Abe, Boerner Yan, Xi, 1990), considered here only.

The scattering matrix is diagonalized by Takagi's theorem (Horn, Johnson, 1985) as is shown in greater detail in (Boerner, Xi, 1990-92):

$$[\,S'\,(AB)\,] = \begin{bmatrix} S'_{AA} & 0 \\ 0 & S'_{BB} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = [\,S_d\,] \tag{12c}$$

$$\lambda_1 = S'_{AA}\,(\rho) = (1 + \rho_1\,\rho_1^*)^{-1}$$
$$(S_{HH} + 2\rho_1\,S_{HV} + \rho_1^2\,S_{VV})\,e^{\,2j\psi_1} = |\,\lambda_1\,|\,e^{\,j\phi_1} \tag{12d}$$

$$\lambda_2 = S'_{BB}\,(\rho_1) = (1 + \rho_1\,\rho_1^*)^{-1}$$
$$(\rho_1^{*2}\,S_{HH} - 2\rho_1^*\,S_{HV} + S_{VV})\,e^{\,2j\psi_4} = |\,\lambda_2\,|\,e^{\,j\phi_2} \tag{12e}$$

The functions of the power returned to the co-pol and cross-pol channels of the receiver are determined from the bilinear form to become:

(i) For the function of the power returned to the cross-pol channel ($\vec{E}_R = \vec{E}_T$) expressed in terms of the antenna height $\vec{h}$ (Boerner, Xi, 1990-92)

$$P_x = |\,V_x\,|^2 = |\,\vec{h}'^{\,T}_{\perp}\,[\,S_d\,]\,\vec{h}'\,|^2 = \frac{1}{\sqrt{1 + \rho'\,\rho'^*}}$$
$$(\,|\,\lambda_1\,|^2\,\rho'\,\rho'^* - \lambda_1\,\lambda_2^*\,\rho'^{*2} - \lambda_1^*\,\lambda_2\,\rho'^2 + |\,\lambda_2\,|^2\,\rho'\,\rho'^*\,) \tag{13a}$$

where ρ' is the polarization ratio of the transceiver in the new basis. The critical points are some $\rho'\,s$ for which the first derivative of P_x with respect to ρ' and ρ'^* vanishes. These critical points, found in function $P_{x'}$ are:

$$\rho'_{xn1} = 0 \qquad \rho'_{xm1,2} = \pm\, i \left(\frac{\lambda_1\,\lambda_2^*}{\lambda_1^*\,\lambda_2} \right)^{1/4} = \pm\, e^{\,j\,(2v + \pi/2)}$$
$$\rho'_{xn2} = \infty$$

$$\rho'_{xs1,2} = \pm \left(\frac{\lambda_1\,\lambda_2^*}{\lambda_1^*\,\lambda_2} \right)^{1/4} = \pm\, e^{\,j2v}\ .$$

(13b)

(ii) For the function of the power returned to the co-pol channel ($\vec{E}_R = \vec{E}_T^{\perp}$)

$$P_c = |V_c|^2 = |\vec{h}_{\perp}'^{\,T} [S_d] \vec{h}'|^2 = \frac{1}{\sqrt{1} + \rho' \rho'^*} (|\lambda_1|^2 + \lambda_1 \lambda_2^* \rho'^{*2} + \lambda_1^* \lambda_2 \rho'^2 + |\lambda_2|^2 \rho'^2 \rho'^{*2}) \tag{14a}$$

the critical points are determined from

$$\rho'_{cm1} = \rho'_{xn1} = 0 \;, \qquad \rho'_{cm2} = \rho'_{xn2} = \infty \;, \tag{14b/c}$$

$$\rho'_{cn1,2} = \pm \left(- \frac{\lambda_1}{\lambda_2} \right) = \pm \left(\frac{|\lambda_1|}{|\lambda_2|} \right)^{1/2} e^{j(2\nu + \pi/2)} \;. \tag{14d}$$

Note that the following conditions are satisfied

$$\rho'_{xn1} \rho'^*_{xn2} = -1 \;, \; \rho'_{xm1} \rho'^*_{xm2} = -1 \;, \; \rho'_{xs1} \rho'^*_{xs2} = -1 \tag{14e/f/g}$$

that means that not only ρ'_{xn1} and ρ'_{xn2} but also ρ'_{xm1} ρ'_{xm2} are 'orthogonal' and so are ρ'_{xs1} and ρ'_{xs2} (Boerner, Xi, 1990-92).

X-POL Null and CO-POL Maximum States

It can be shown for the monostatic reciprocal case that the X-POL Nulls and the CO-POL Maxima are identical (Boerner, Xi, 1990; Xi, Boerner, 1992) as shown in (14e,f). The power returns to the cross/co-pol channels are (Boerner, Xi, 1990-92)

$$P_{xn1}(\rho'_{xn1}) = P_{xn2}(\rho'_{xn2}) = 0 \;, \; P_{co1}(\rho'_{cm1}) = |\lambda_1|^2 \;, \; P_{co2}(\rho'_{cm2}) = |\lambda_2|^2 \,. \tag{15a}$$

CO-POL Nulls, X-POL Maxima and X-POL Saddles

The $\rho'_{xm1,2}$ of (13) are the cross-pol maxima and $\rho'_{xs1,2}$ of (13) the cross-pol saddles. The corresponding power returns to the receiver of the cross/co-pol channels are (Fig.2a):

$$P_x(\rho_{xm1,2}) = \frac{1}{4}(|\lambda_1| + |\lambda_2|)^2$$

$$P_c(\rho_{xm1,2}) = \frac{1}{4}(|\lambda_1| - |\lambda_2|)^2 \tag{15b}$$

$$P_x(\rho'_{xs1,2}) = \frac{1}{4}(|\lambda_1| - |\lambda_2|)^2$$

$$P_c(\rho'_{xs1,2}) = \frac{1}{4}(|\lambda_1| + |\lambda_2|)^2$$

The $\rho'_{cn1,2}$ of (15) are the co-pol nulls which may be considered to be 'pseudo-extrema' (Mieras, 1983, Mieras, Barnes, Vachula, Bucknam, Boerner, 1982), because the power returned to the co-pol channel becomes zero, (Kennaugh, 1992-81; Agrawal, Boerner, 1989-92; Boerner, Xi, 1990-92; Yan, Boerner, 1991, Boerner, Yan, Xi, 1992), i.e.,

$$P_c(\rho'_{cn1,2}) = 0 \tag{15c}$$

3.2.2 The Polarization Fork [6,7,17]

In order to determine the polarization fork (Boerner, Xi, 1990-92), use is made of the polarization ratio ρ formulation of (1b), shown in [Fig.1c], relating point (ρ'_s) on the complex plane with $P(\rho'_s)$ on the Poincaré) sphere (Boerner, Xi, 1990-92). According to (1b), each point ρ′ of the complex plane can be connected to the Zenith (LC) of the sphere, resting tangent to the complex plane in its origin O at the Nadir (RC), by a straight line that intersects the sphere at one point $P(\rho'_s)$, where the Nadir (RC) corresponds to the origin (O) of the plane, the Zenith (z) to the circle at "infinity (∞)", and the equator to the unit circle, representing linear polarization states (Boerner, Xi, 1990-92) as illustrated in Fig. 2a. According to the expression of the cross-pol max and cross-pol saddles, they all lie on the unit circle and are the end points of two orthogonal diameters. So their corresponding points lie on the equator of the sphere as S_1,S_2,T_1 and T_2 with S_1 S_2 and T_1 T_2 perpendicular (at right angles on the polarization sphere) to each other. The co-pol nulls of $\rho'_{scn1,2}$ lie on the same straight line with $\rho'_{sxm1,2}$ on the plane and symmetric about the origin O, so their corresponding points on the sphere C_1 and C_2 lie on the same great circle with X_1,X_2,S_1 and S_2 symmetric about the diameter X_1 X_2, also denoted as '**KennaughGs Characteristic Circle (Plane)**'(Fig.2b).

3.2.3 Huynen's Presentation [2,7]

Using Huynen's geometric parameters, the properties of the scattering matrix [S], can be expressed according to $[S'(AB)] = [U]^T[S][U]$, as (Boerner, Xi, 1990-92)

$$[S] = [U^*(\rho_1)] \exp(\nu [L]^*) \, m \begin{bmatrix} 1 & 0 \\ 0 & \tan^2\gamma \end{bmatrix} \exp(\nu [L]^*)^T [U^*(\rho_1)]^T \exp(j\xi) \tag{16a}$$

$$[U^*](\rho_1)] = \frac{1}{\sqrt{1 + \rho_1 \rho_1^*}} \begin{bmatrix} e^{-j\psi_1} & -\rho_1(\phi_m, \tau_m) e^{-j\psi_4} \\ \rho_1^*(\phi_m, \tau_m) e^{-j\psi_1} & e^{-j\psi_4} \end{bmatrix} \tag{16b}$$

which is, as shown in (Boerner, Xi, 1990-92) and illustrated in Fig. 2c, the same as Huynen's [H] given by

$$[H] = [U^*(\psi, \tau_m, \nu)]m \begin{bmatrix} 1 & 0 \\ 0 & \tan^2\gamma \end{bmatrix} U'^*(\psi, \tau_m, \nu)]\exp(j\xi) \tag{16c}$$

and $[U(\psi, \tau_m, \nu)] = e^{\psi[J]} e^{\tau_m[K]} e^{\nu[L]}$, (16d)

where j[J], j[K] and j[L] are related to the Pauli spin matrices $[\rho_i]$ with i = 1,2,3; and [I] is the identity matrix ($[I] = [\rho_0]$) defined in Section 7. Huynen's parameters [2] m, ϕ_μ,ν,γ,δ_m and $\alpha_{m'}$ are defined in Fig. 2c. The identity of (16b) with (16d) establishes an important new matrix transformation identity on the polarization sphere, in that the various sets of optimal polarization states can be straight-forwardly related to one another as was first derived in (Boerner, Xi, 1990).

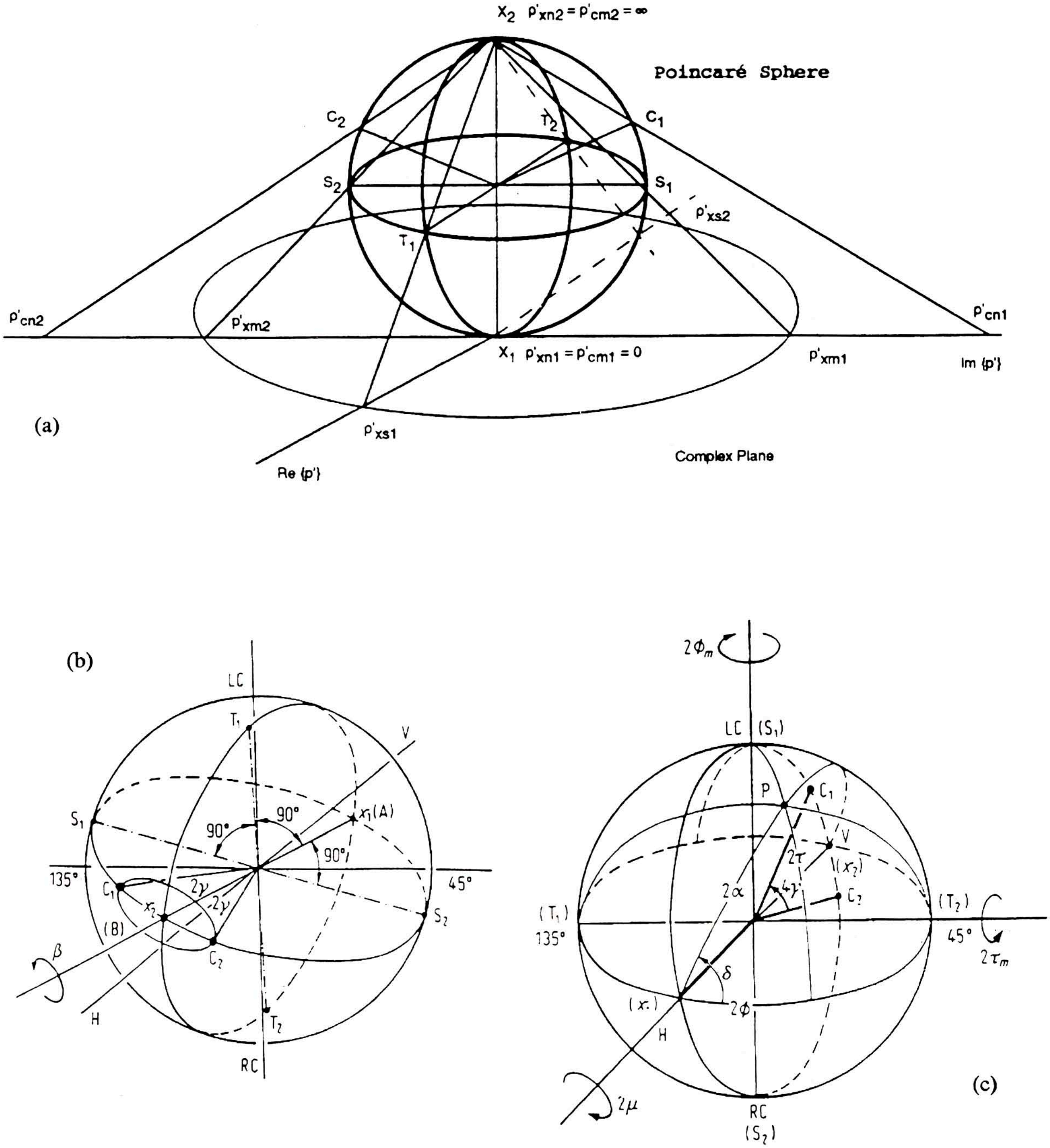


Fig. 2 - POLARIZATION FORK: (a) Correspondence of $\rho_{xn1,2}' = \rho_{cm1,2}' = \rho_{xm1,2}' = \rho_{xs1,2}'$ and $\rho_{cn1,2}$ *on the complex plane with* $X_{1,2}$,$T_{1,2}$,$S_{1,2}$, and $C_{sub}1,2$, *respectively, on the Poincaré sphere ; (b) Representation of the characteristic polarization states on the Poincaré sphere (*X_1*, cross-pol null and co-pol max ;* X_2*, cross-pol null and co-pol extremum ;* $C_{1,2}$*, co-pol nulls ;* $S_{1,2}$*, cross-pol max;* $T_{1,2}$*, cross-pol saddle points;* γ*, target characteristic angle) presented in the new basis (AB) ; (c) Standardized polarization fork of Huynen with definition of Huynen's geometrical parameters presented in the old basis .* ϕ *- target orientation or tilt angle;* ν *- target skip angle;* τ *- target ellipticity angle;* γ *- target characteristic angle;* ρ *- tan* α *exp(j*δ*), polarization ratio.*

3.3 Optimization Approach Using Stokes Vector and Mueller Matrix Formalism: $[M]$, $[\overline{M}_c]$, $[\overline{M}_x]$ and $[\overline{M}_m]$

Using the Lagrange multipliers method applied to the received power expressed in terms of the Stokes reflection matrices $[\overline{M}_c]$, $[\overline{M}_x]$ and $[\overline{M}_m]$ of (6d),(6e) and (6f), respectively; this method, derived in (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992), enables one to obtain characteristic polarization states for the symmetric (reciprocal), a symmetric (nonreciprocal), monostatic and bistatic cases. The components of $\vec{g_T}$ satisfy the following constraint equation

$$\phi\,(g_{T1}, g_{T2}, g_{T3}) = \sqrt{g_{T1}^2 + g_{T2}^2 + g_{T3}^2} - 1 = 0 \tag{17a}$$

for which the Lagrangian multipliers method for determining the extrema of the power $P\,(g_{T1}, g_{T2}, g_{T3}$ results in

$$\frac{\partial P}{\partial g_{Ti}} - \mu \frac{\partial \Phi}{\partial g_{Ti}} = 0 \ , \quad (i = 1,2,3) \ . \tag{17b}$$

The corresponding optimal polarization states, corresponding to the "degenerated" Mueller matrix $[\overline{M}]$, the Stokes reflection, (Kennaugh) matrices $[\overline{M}_c]$, $[\overline{M}_x]$ and to $[\overline{M}_m]$, are obtained by solving the corresponding normalized power expressions P_c, P_x and P_m of (6c), (6d) and (6e), respectively in (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Yamaguchi, Sasagawa, Sengoku, Abe, Boerner, Yan, Xi, 1990; Boerner, Liu, Zhang, 1992). It should be noted here that these Mueller matrix optimization approaches implementing the Lagrange multipliers method in radar polarimetry were first initiated at UIC-EECS/CSL (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Kostinski, James, Boerner, 1988; Tanaka, Boerner, 1992) and independently developed using alternate formulations also by Van Zyl (van Zyl, 1986, van Zyl, Papas, Elachi, 1987, Zebker, van Zyl, 1991) and others (Cloude, 1986-92-90-91-88).

3.4 Optimization Approach for the Covariance Matrix Method: $[\Sigma\,(HV = VH)]$

The optimal polarization states associated with the covariance matrix $[\Sigma\,(HV)]$, of eq. (4), can be obtained by decomposing the 3x3 unitary transformation matrix $[\Upsilon(\rho)]^+$ of eq. (4e) into its two-parameter complex column normalized vectors $\vec{z_i}(\rho)$ according to (Boerner, Liu, Zhang, 1992; Tragl, 1992-90, Tragl, LGneburg, Schroth, Ziegler, 1991), as

$$[\Upsilon(\rho)]^+ = [\,\vec{z_1}(\rho)\,\vec{z_2}(\rho)\,\vec{z_3}(\rho)\,] \tag{18a}$$

with

$$\vec{z_1}(\rho) = \frac{1}{1+\rho\rho^*}\begin{bmatrix} 1 \\ \sqrt{2}\,\rho^* \\ \rho^{*2} \end{bmatrix}, \quad \vec{z_2}(\rho) = \frac{1}{1+\rho\rho^*}\begin{bmatrix} -\sqrt{2}\,\rho \\ 1-|\rho|^2 \\ \sqrt{2}\,\rho^* \end{bmatrix},$$

$$\vec{z_3}(\rho) = \frac{1}{1+\rho\rho^*}\begin{bmatrix} \rho^2 \\ -\sqrt{2}\,\rho \\ 1 \end{bmatrix},$$

where the $\vec{z_i}(\rho)$ are associated with the mean power expressions for the co-polar and cross-polar channels according to (9) as:

$$P_c(\rho) = \vec{z_1}(\rho)^+ \,[\,\Sigma\,(HV)\,]\, \vec{z_1}(\rho) \tag{18b}$$

$$P_x(\rho) = (1/2)\, \vec{z_2}(\rho)^+ \,[\,\Sigma\,(\rho)\,]\, \vec{z_2}(\rho) \tag{18c}$$

$$P_c^{\perp}(\rho) = \vec{z_3}(\rho)^+ \,[\,\Sigma\,(\rho)\,]\, \vec{z_3}(\rho) \tag{18d}$$

The existence of extrema as it relates to the covariance matrix power expressions (18b,c,d) is guaranteed by the Weierstrass theorem (Tragl, 1992-90, Tragl, LGneburg, Schroth, Ziegler, 1991) applied to the compactness of the set of all possible optimal polarization states on the Poincaré) polarization sphere and the continuity of the power expressions. The extremal values of the respective power functions $P_c(\rho)$, $P_c(\rho)$ and $P_x(\rho)$ can be determined by equating the first derivatives with respect to ρ^* to zero: $\partial\{P(\rho, \rho^*)\}/\partial\rho^* = 0$. The solutions can be calculated by regarding the power function to depend on the two independent variables ρ and ρ^*

$$\frac{\partial P_c(\rho, \rho^*)}{\partial \rho^*} = \frac{2}{(1+\rho\rho^*)} R_x(\rho, \rho^*) = 0 \ , \tag{19a}$$

$$\frac{\partial P_c^{\perp}(\rho, \rho^*)}{\partial \rho^*} = -\frac{2}{(1+\rho\rho^*)} R_x^{\perp}(\rho, \rho^*) = 0 \ , \tag{19b}$$

$$\frac{\partial P_x(\rho, \rho^*)}{\partial \rho^*} = \frac{1}{(1+\rho\rho^*)} (R_x^{\perp}(\rho, \rho^*) - R_x(\rho, \rho^*)) = 0 \ . \tag{19c}$$

From equations (9) and (18), it follows that the copolar power density optimum corresponds to a vanishing degree of coherence μ_{AB}. In other words, if a copolar power maximum (co-pol max) is transmitted, then the backscattered orthogonal wave components are mutually incoherent and (9f) reduces to

$$q_{AB}\,(\rho_{co-\max}) = \frac{|P_c - P_x|}{P_c + P_x} \tag{19d}$$

These three extremal conditions can be solved by standard numerical techniques either directly in the complex plane or by reducing the problem to that of the numerical solution of two coupled non-linear equations by the separation of the real from the imaginary parts resulting in a truly tedious and cumbersome exercise. Instead, we are going to employ a more elegant method, recently developed by Lüneburg et al. in (LGneburg, Ziegler, Tragl, Schroth, 1991) by introducing a set of unconstrained real vectors $\vec{v}$ which need to be determined separately for the co-polar and cross-polar power optimization approaches of (18) using a different formulation for the two channels (Yan, Boerner, 1992-91, Boerner, Yan, Xi, 92). Another optimization approach in terms of the optimal target substructure matrices derived from the target matrix decomposition is presented in (Cloude, 1991) and its results need to be fused with ours, i.e., proved to be identical

3.3.1 Cross-polar Power $P_X(\rho)$ Optimization

In the cross-polar power case, an unconstrained real vector $\vec{v}(\rho)$ is derived from the complex vector $\vec{z_2}(\rho)$ of (15) via a ρ - independent linear unitary transformation matrix [Q] such that

$$\vec{v}(\rho) = [\,Q\,]\,\vec{z}(\rho) = \frac{1}{(1+\rho\rho^*)}\begin{bmatrix} 1-\rho\rho^* \\ 2Re\rho \\ 2Im\rho \end{bmatrix},$$

$$[\,Q\,] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & \sqrt{2} & 0 \\ -1 & 0 & 1 \\ j & 0 & j \end{bmatrix} \qquad (20a)$$

Due to the orthogonality relation $\rho\rho^* = -1$ according to (1), the solution for ρ and for the orthogonal polarization state $\rho = (-1/\rho^*)$ will provide $\vec{v}(\rho)$ and $-\vec{v}(\rho)$; i.e, vectors differing in sign only.

$$P_X(\rho) = \frac{1}{2}\vec{z_2}^+(\rho)\,[\,\Sigma\,(HV)\,]\,\vec{z_2}(\rho) = \frac{1}{2}\vec{v}(\rho)^T\,[Q]\,[\,\Sigma\,(HV)]$$

$$[\,Q\,]^+\,\vec{v}(\rho) = \frac{1}{2}\vec{v}(\rho)^T\,[\,\Lambda\,(HV)\,]\,)\,\vec{v}(\rho) = \qquad (20b)$$

$$= \frac{1}{2}\vec{v}^T(\rho)\,Re\,(\,[\,\Lambda\,(HV)\,]\,)\,\vec{v}(\rho)$$

with the Hermitian alternate covariance matrix $[\,\Lambda\,(HV)\,]$ given by

$$[\,\Lambda\,(HV)\,] = [\,Q\,]\,[\,\Sigma\,(HV)\,]\,[\,Q\,]^+ = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{bmatrix}, \qquad (20c)$$

$$\Lambda_{11} = 2 < |\,S_{HV}|^2 > ; \; \Lambda_{12} = \{ < S_{HH}S_{VV}^* > - < S_{HV}S_{HH}^* > \};$$

$$\Lambda_{13} = -j\,\{ < S_{HV}S_{HH}^* > + < S_{HV}S_{VV}^* > \};$$

$$\Lambda_{21} = \{ < S_{VV}S_{HV}^* > - < S_{HH}S_{HV}^* > \};$$

$$\Lambda_{22} = \frac{1}{2}\{ < |\,S_{HH}|^2 > + < |\,S_{VV}|^2 > - < S_{HH}S_{VV}^* > - < S_{VV}S_{HH}^* > \};$$

$$\Lambda_{23} = \frac{1}{2}j\,\{ < |\,S_{HH}|^2 > - < |\,S_{VV}|^2 > + < S_{HH}S_{VV}^* > -$$

$$+ < S_{VV}S_{HH}^* > \};\; \Lambda_{31} = j\,\{ < S_{HH}S_{HV}^* > + < S_{VV}S_{HV}^* > \}; \qquad (20d)$$

$$\Lambda_{32} = -\frac{1}{2}j\,\{ < |\,S_{HH}|^2 > - < |\,S_{VV}|^2 > + < S_{VV}S_{HH}^* > -$$

$$+ < S_{HH}S_{VV}^{2} > \};$$

$$\Lambda_{33} = \frac{1}{2}\{ < |\,S_{HH}|^2 > + < |\,S_{VV}|^2 > + < S_{HH}S_{VV}^* > +$$

$$+ < S_{VV}S_{HH}^* > \}$$

with

$$Trace\,(\,[\Lambda(HV\,)]) = Trace([\Sigma(HV\,)\,]\,) = Span(\,<[S(HV)>\,) \qquad (20e)$$

subject to the constraint (Boerner, Liu, Zhang, 1992; Tragl, 1992-90-91)

$$\vec{v}^T\,\vec{v} = 1 \qquad (20f)$$

The solutions are found from applying the standard method $|\,Det\{\,[\,\Lambda\,(HV)\,]\,\} - v'\,[\,I\,]\,| = 0$, where the real eigenvectors $\pm\vec{v}$ can be associated with the solution Stokes vectors $\vec{g}$ as

$$\vec{g_i} = (1 + \vec{v_i}^T) \quad \text{and} \quad \vec{g_{i_\perp}} = (1 - v_i^T)\;,\; \vec{g_i}\cdot\vec{g_\perp} = 0 \qquad (20g)$$

and $\vec{g_{i_\perp}}$ denotes the orthogonal polarization state (antipodal on the Poincaré) sphere). The resulting solution is identical to that obtained from optimizing $P_x(\rho)$ given by (6d) using the Mueller matrix optimization method (Yan Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan Xi, Yamaguchi, 1992).

3.3.2 Co-polar Power Optimization (Covariance Matrix Approach)

Applying the -independent linear unitary transformation matrix [Q] of (20) to the complex column $z_1(\rho)$, yields

$$[\,Q\,]\,\vec{z_1}(\rho) = \frac{1}{\sqrt{2}}\,[\,\vec{x}(\rho) + j\,\vec{\gamma}(\rho)\,] \qquad (21a)$$

with

$$\vec{x}(\rho) = (2Re\rho\,,\,Re^2\rho - Im^2\rho - 1\,,\,2\,Re\,\rho\,Im\,\rho)^T/(1 + |\,\rho\,|^2)$$

$$\vec{y}(\rho) = (-2Im\,\rho\,,\,-2Re\rho\,Im\,\rho\,,\,Re^2\rho - Im^2\rho + 1)^T/(1 + |\rho|^2)$$

so that the co-polar power function $P_c(\rho)$ becomes

$$P_c(\rho) = \vec{z_1}(\rho)^+\,[\Sigma\,(HV)\,]\vec{z_1}(\rho) = \frac{1}{2}\vec{x}^T(\rho)\,Re(\,[\Lambda\,(HV)\,]\,\vec{x}(\rho) \qquad (21b)$$

$$+\frac{1}{2}\vec{\gamma}^T(\rho)\,Re(\,[\Lambda\,(HV)\,]\,\vec{\gamma}^T(\rho)\frac{1}{2}\vec{\gamma}^T(\rho)\,Im(\,[\Lambda\,(HV)\,]\,\vec{x}(\rho)$$

where
$\mathrm{Im}\{[(\Lambda(hv)]\} = -\frac{1}{2}j\{[\Lambda(HV)] - [\Lambda(HV)]^T\}$ is real and anti-symmetric (Tragl, 1992-90-91).

Introducing the auxiliary vector $\vec{b_0} = (b_{01}\ b_{02}\ b_{03})^T$, where

$$b_{01} = \frac{1}{2}(<|S_{HH}|^2> - <|S_{VV}|^2>) = \mathrm{Im}\{\Lambda_{23}\}, \quad (21c)$$

$$b_{02} = Re(<S_{HV}S^*_{VV}> + <S_{HH}S_{HV}>) = \mathrm{Im}\{\Lambda_{31}\}$$

$$b_{03} = \mathrm{Im}(<S_{HV}S^*_{VV}> + <S_{HH}S^*_{HV}>) = \mathrm{Im}\{\Lambda_{12}\}$$

it can be shown by using the geometrical vector product (Tragl, 1992-90-91)

$$\vec{x}(\rho) \times \vec{b_0} = \mathrm{Im}\{[(HV)]\}\vec{x}(\rho) \quad (21d)$$

that

$$\vec{\gamma}^T \mathrm{Im}\{[\Lambda]\}\vec{x} = \vec{\gamma}\cdot(\vec{x}\times\vec{b_0}) = (\vec{\gamma}\times\vec{x})\cdot\vec{b_0} = \vec{v}^T\vec{b_0} \quad (21e)$$

since

$$\vec{v}(\rho) = \vec{\gamma}(\rho)\times\vec{x}(\rho) = \{(1-\rho\rho^*)\ 2Re\rho\ \ 2Im\rho\}^T. \quad (21f)$$

Introducing the real orthogonal transformation matrix $[O](\rho)]$ with column vector $\vec{x}(\rho)$, $\vec{\gamma}(\rho)$ and $\vec{v}(\rho)$

$$[O(\rho)] = [\vec{x}(\rho)\vec{\gamma}(\rho)], \quad [O][O]^T = [I], \quad (22a)$$

it can be shown (Tragl, 1992-90-91) that

$$Trace([O(\rho)]^T Re([\Lambda(HV)])[O(\rho)]) = $$
$$= Trace(Re[\Lambda(HV)])$$
$$(22b)$$
$$= \vec{x}^T Re([\Lambda(HV)])\vec{x} + \vec{\gamma}^T Re([\Lambda(HV)])\vec{\gamma} + \vec{v}^T Re([\Lambda(HV)])\vec{v}$$

and that with

$$[B(HV)] = \frac{1}{2}(Trace(Re([\Lambda(HV)])[I] - Re([\Lambda(HV)])) \quad (22c)$$

another expression for $P_c(\rho)$ is obtained as

$$P_c(\rho) = \vec{v}^T(\rho)[B(HV)]\vec{v}(\rho) + \vec{b}^T(HV)\vec{v}(\rho) \quad (22d)$$

$$\text{which with } \vec{g_\pm}(\rho) = (1 \pm \vec{v}^T(\rho))^T \quad (22e)$$

is shown to be identical to the expressions obtained directly for the co-polar Mueller matrix power expression $P_c(\rho)$ of (6c). For details concerning optimization procedures in the context to the covariance matrix optimization approaches we refer to (Boerner, Liu, Zhang, 1992; Tragl, 1992-90-91). Following the same procedure of using the Lagrangian multiplier method developed first in (Kostinski, James, Boerner, 1988), it can be shown that the real vector $\vec{v}(\rho)$, defined in (20) and satisfying (21g), which extremizes the quadratic form for $P_C(\rho)$ of (22d) are solutions of the set of coupled non-linear equations in $\vec{v}$ and

$$\{[B(HV)]\vec{v}(\rho) - \mu'\vec{v}(\rho)\} = -\frac{1}{2}\vec{b}(HV), \quad \vec{v}(\rho)^T\vec{v}(\rho) = 1. \quad (22f)$$

In conclusion, the optimizing solutions obtained via the corrected polarimetric covariance matrix method for the coherent case are identical to those obtained via the 'Mueller Matrix' (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992) and the 'Critical Point' (Boerner, Xi, 1990-92) or 'Basis Transformation' (Agrawal, Boerner, 1989-92) optimization methods as is shown in (Boerner, Liu, Zhang, 1992; Boerner, Yan, Xi, Yamaguchi, 1992). These alternate optimization results obtained for the symmetric coherent case present a very important contribution to radar polarimetry in that a possible approach for solving the optimization procedures in closed form also for the partially polarized and partially coherent cases may have been pioneered (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Yamaguchi, Sasagawa, Sengoku, Abe, Boerner, Yan, Xi, 1990; Boerner, Liu, Zhang, 1992; Tragl, 1992-90-91; Boerner, Yan, Xi, Yamaguchi, 1992) by first treating the coherent case for which analytical solutions for P_x and P_c exist.

4. OPTIMAL POLARIZATION STATES FOR THE PARTIALLY POLARIZED CASE

Consider a time-independent scatterer which is illuminated by a monochromatic (completely polarized) wave $\vec{E_T}$, for which the reflected wave $\vec{E_S}$ is, in general, non-monochromatic; and therefore, partially polarized. Consequently, the Stokes vector and Mueller matrix formalism will be employed. There are three types of energy density terms, next to the total energy density g_{S0}, that can be optimized according to (7b,c,d) in Section 2.5. We note here that alternate approaches were introduced by Van Zyl (van Zyl, 1986-87, Zebker, van Zyl, 1991) and Cloude (Cloude, 1988) which are based on principles of target decomposition (Huynen, 1965-78-92-90, Huynen, McNulty, Hanson, 1975; Boerner, 1992), but not further considered here.

4.1 Optimization of the adjustable intensity $q\,g_{S0}$

The energy density qg_{S0}, contained in the completely polarized part $\vec{g_q}$, is called the adjustable intensity because one may adjust the polarization state of the receiver to ensure the polarization match. We can rewrite the scattering process (7) in index notation as (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Yamaguchi, Sasagawa, Sengoku, Abe, Boerner, Yan, Xi, 1990; Giuli, 1986):

$$g_{Si} = \sum_{j=0}^{3} M_{ij}\, g_{Tj}\,, \tag{23a}$$

where j = 0,1,2,3. The adjustable intensity qg_{S0} has the following property:

$$qg_{S0} = \left(\sum_{i=0}^{3} g_{Si}^2 \right)^{1/2} = \left[\sum_{j=0}^{3} \left(\sum_{j=0}^{3} M_{ij}\, g_{Tj} \right)^2 \right]^{1/2} \tag{23b}$$

where g_{Ti}'s are the elements of the Stokes vector of the transmitting wave Yan, Boerner, 1991, Boerner, Yan, Xi, 1992). The partial derivative of $(qg_{S0})^2$ with respect to g_{Tk} can be derived as:

$$\frac{\partial\,(qg_{S0})^2}{\partial\, g_{Tk}} = \sum_{i=1}^{3} \frac{\partial\, g_{Si}^2}{\partial\, g_{Tk}} = 2 \sum_{i=1}^{3} g_{Si}\, M_{ik} = 2 \sum_{i=1}^{3} \sum_{j=0}^{3} M_{ij}\, M_{ik}\, g_{Tj}\ . \tag{23c}$$

For optimizing the adjustable intensity, we apply the method of Lagrangian multipliers (Leitman, 1962), which yields

$$\frac{\partial\,(qg_{S0})^2}{\partial\, g_{Tk}} - \mu\, \frac{\partial\, \Phi}{\partial g_{Tk}} = 2 \sum_{i=1}^{3} \sum_{j=0}^{3} \left(M_{ij}\, M_{ik}\, g_{Tj} - \mu\, g_{tk} \right) = 0\ , \tag{23d}$$

where Φ, is the constraint equation of (17a). Equation (23c) is a set of inhomogeneous linear equations in $g_{T1}(\mu)$, $g_{T2}(\mu)$ and $g_{T3}(\mu)$. Then, the straightforward solutions for the $g_{Ti}(\mu)$ are three functions of μ. Substituting $g_{Ti}(\mu)$, $(i = 1,2,3)$ into the constraint condition of (17a) leads to a sixth-order polynomial equation of μ (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Bostinski, James, Boerner, 1988). For each μ value, we calculate $g_{T1}\, g_{T2}\, g_{T3}$, and qg_{S0} according to the formulae in (23a). The largest (or smallest) intensity is the optimal intensity, the corresponding $\vec{g_T}$ is the optimal polarization state of the transmitted wave (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992).

4.2 Minimizing the noise-like energy density term: $(1 - q)\, g_{S0}$

An unpolarized wave can always be represented by an incoherent sum of any two orthogonal completely polarized waves of equal intensity (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992), which leads to 50% efficiency for the reception of the unpolarized part of the scattered wave given by:

$$(1 - q)\, g_{S0} = g_{S0} - qg_{S0} = \sum_{j=0}^{3} M_{0j}\, g_{Tj} - + \sqrt{\sum_{i=1}^{3} \left(\sum_{j=0}^{3} M_{ij}\, g_{Tj} \right)^2} \tag{24}$$

Hitherto, no simple method was found giving the analytic closed form solution for the minimum solution [8], instead, a computer numerical analysis was used, although it looks feasible to find the desirable closed-form solution using an alternate Newton-Kantorovich minimization method (Leitman, 1962; Wait, 1979) also discussed in (Cloude, 1991).

4.3 Maximizing the receivable intensity in the scattered wave: $(1 - q)\, g_{S0}$

The total receivable energy density consists of two parts: 100% reception efficiency for the completely polarized part of the scattered wave and; 50% reception efficiency for the unpolarized part. We may write the following expression for the total receivable intensity:

$$\frac{1}{2}(1 + q)\, g_{S0} = qg_{S0} + \frac{1}{2}(1 - q)\, g_{S}S0 = \frac{1}{2} \sum_{j=0}^{3} M_{0j}\, g_{Tj} + \frac{1}{2} \sqrt{\left(\sum_{i=1}^{3} \sum_{j=0}^{3} M_{ij}\, g_{Tj} \right)^2}\ . \tag{25}$$

Also, this equation can only be solved using numerical analysis, unless we succeed with implementing the alternate Newton-Kantorovich method (Cloude, 1991; Leitman, 1962; Wait, 1979) successfully.

5. NUMERICAL EXAMPLE AND INTERPRETATION OF MATRIX OPTIMIZATION RESULTS

We have demonstrated that there exist several different approaches for determining the optimal polarization states in coherent monostatic radar polarimetry yielding the identical results as illustrated for one example (Table 1) in Fig.3 and Fig.4 showing and proving that Kennaugh's target characteristic theory and Huynen's polarization fork concept for the "coherent symmetric case" are correct and valid (which was set in doubt in various unpublished government agency reports). For the partially coherent case, no complete optimization procedure to determine the optimal polarization states yet exists. However, from a comparison of our results (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Yamaguchi, Sasagawa, Sengoku, Abe, Boerner, Yan, Xi, 1990; Boerner, Liu, Zhang, 1992;

Boerner, Yan, Xi, Yamaguchi, 1992; Kostinski, James, Boerner, 1988), we may conclude that the optimal polarization state theory will also be highly useful for treating the partially dual polarization radar reception problem as treated in (Yan, Boerner, 1991, Boerner, Yan, Xi, 1992; Boerner, Yan, Xi, Yamaguchi, 1992). In all of the cases investigated, it was demonstrated that for the partially polarized case there also seem to exist five pairs of characteristic polarization states (Boerner, Yan, Xi, Yamaguchi, 1992); however, whereas, for the coherent case (q = 1) the absolute (normalized) power maximum at the co-pol max (ρ_{cm1}) and co-pol null ($\rho_{cn1,2}$) locations, respectively, becomes

$$P^c_{\max}(\rho_{cm1})/m^2 = 1 \ , \quad P^c_{cn1,2}(\rho_{cn1,2})/m^2 = 0 \ , \qquad (26)$$

we find that for the partially polarized case ($0 < q < 1$) the maximum normalized value will always be reduced by (1 - q)/2 and the achievable minimal normalized power can never be less than (1 - q)/2, and that according to (5e) for the completely unpolarized case (q = 0), the achievable minimal and maximal normalized powers become equal and in the limit approach $g_{S0} = 0.5$; i.e., the power density plot is flat in the extreme unpolarized case as illustrated in Fig.5. In order to further pursue this heuristic finding, first the direct relation to the eigenvalue/vector properties of the 'Corrected Polarimetric Covariance Matrix', as expressed in (10a), must be established separately for the co-polarized and cross-polarized power density expressions by determining the upper and lower bounds directly in terms of the eigenvalues ν_i of (10a), In a next step, direct optimization procedures of the Pauli spin matrix [σ_i ; $i = 0,1,2,3$] formulation of the Mueller matrix and the covariance matrix approach (Tragl, 1992-90, Tragl, LGneburg, Schroth, Ziegler, 1991) need to be further advanced together with SU(n = 2,3,4) Lie group theoretic analyses implementing next to the 2×2 Pauli spin matrices also the 3×3 Hausdorff (or alternate Gell-Mann) [δ_i ; $i = 1,2,...,9$] and the 4×4 Dira [θ_i ; $I = 0,1,2,...,15$] matrices [10,20], so that the true analytic expression for the respective power density plots may also be established for the partially polarimetric cases. The heuristic finding displayed in Fig.5 can also be closely associated with the 'polarimetric target decomposition' approaches of Huynen (Huynen, 1965-78-92-90, Huynen, McNulty, Hanson, 1975) and its alternate presentations of Barnes (Barnes, 1984), Holm (Holm, Barnes, 1992), Pottier (Pottier, 1990), and especially Cloude (Cloude, 1986-92-90-91-88), which still represents one of the main unanswered questions in radar polarimetry. However, after having established the unique relations existing between [S], [G], [Σ] and [M_i] for the coherent case, the proper treatment and correct evaluation of Huynen's major contribution to radar polarimetry (Huynen, 1965-78-92-90, Huynen, McNulty, Hanson, 1975), i.e., his target matrix composition into sub-target-structure matrices, may now be accomplished.

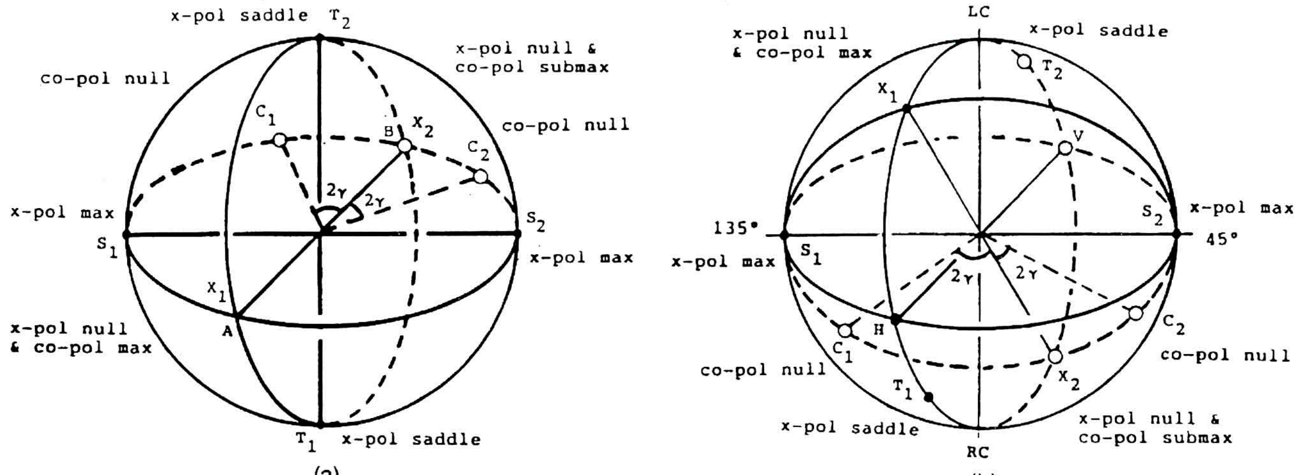


Fig. 3 - DISPLAY OF POLARIZATION FORK FOR SCATTERING MATRICES OF TABLE 1: (a) Characteristic polarization states on the Poincaré sphere of the Example referenced to the new basis (AB) for the scattering matrix [S] with the characteristic polarization ratios $\rho_{xn1}' = \rho_{cm1}' = 0$ (X_1), $\rho_{xn2}' = \rho_{cm2}' = \infty$ (X_2), $\rho_{xm1,2}' = \pm$ $\pm j$ (S_1,S_2), $\rho_{xs1,2}' = \pm 1$ (T_1,T_2), and $\rho_{cn1,2}' = \pm 1.668j$ (C_1,C_2); *(b) Characteristic polarization states on the Poincar) referenced to the old basis (HV) for the scattering matrix [S] of the example with the characteristic polarization ratios*

$\rho_{xn1}' = \rho_{cm1}' = 0.414 \exp(j90^0)$ (X_1), $\rho_{xn1} = \rho_{cm2} = 2.414 \exp(-j159^0)$ (C_1), and $\rho_{cn2} = 1.414 \exp(-j20.7^0)$ (C_2)
and the geometric parameters $\nu = 0.0$, $\gamma = 30.9$, $\delta_m = 90.0$, $\alpha_n = 22.5$, $\phi = 0.0$, and $\tau_m = 22.5$.

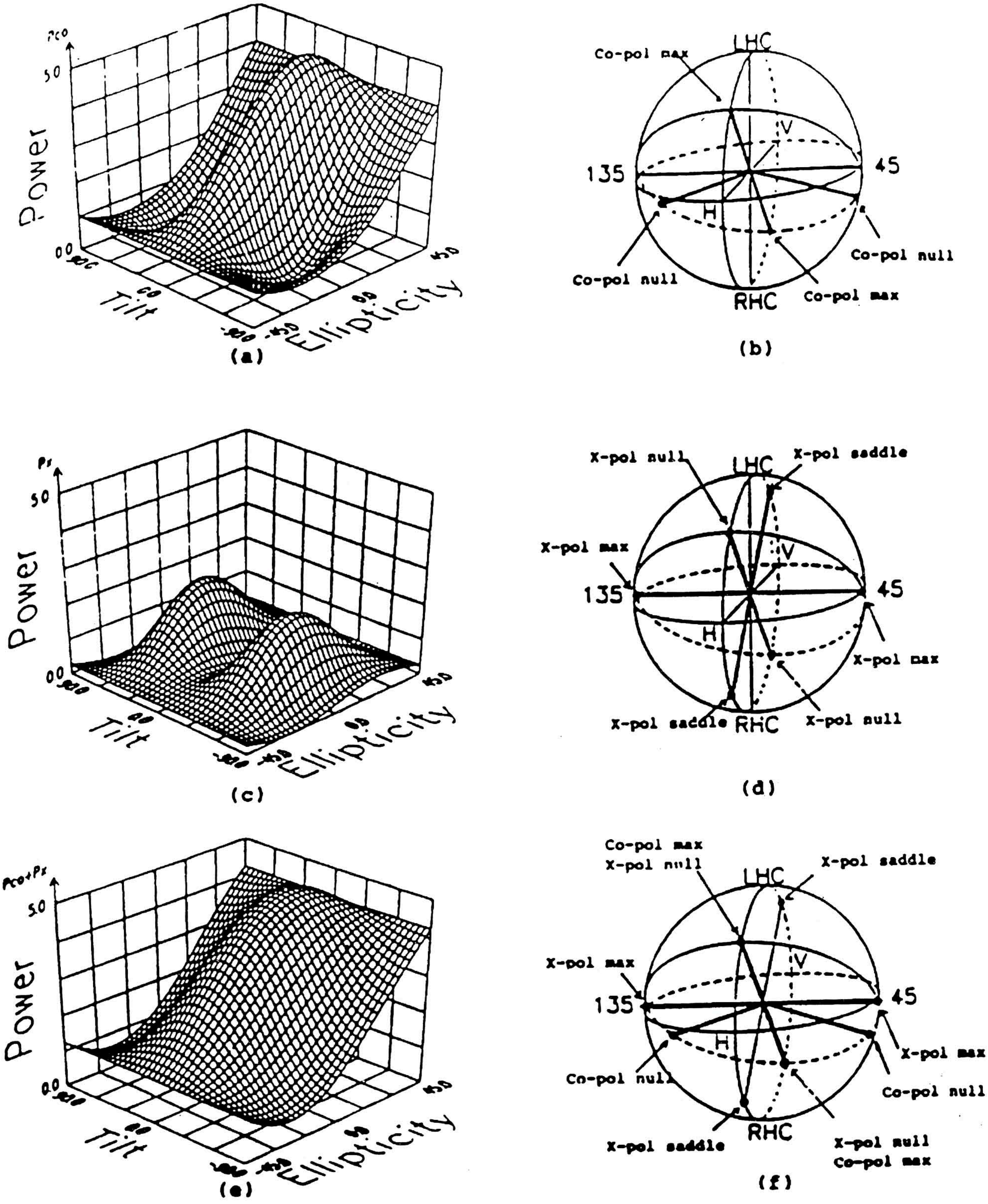
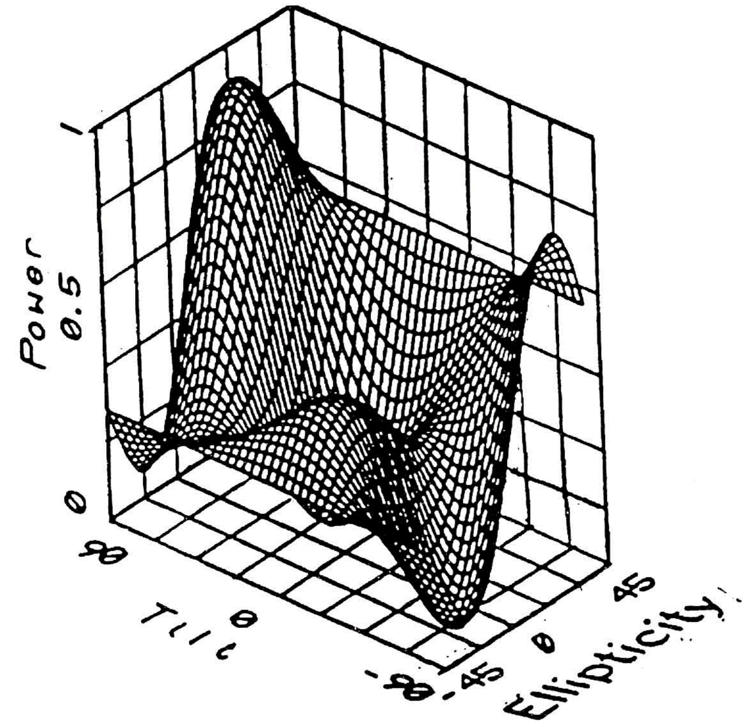


Fig. 4 - POLARIZATION STATE CHARACTERISTICS FOR [S], [Σ], AND [M] FOR COHERENT CASE OF TABLE 1: (a) Co-polarized power spectrum; (b) Co-polarization states; (c) X-polarized power spectrum; (d) X-polarization states; (e) Power spectrum for matched two-antenna case.

(a)



(b)

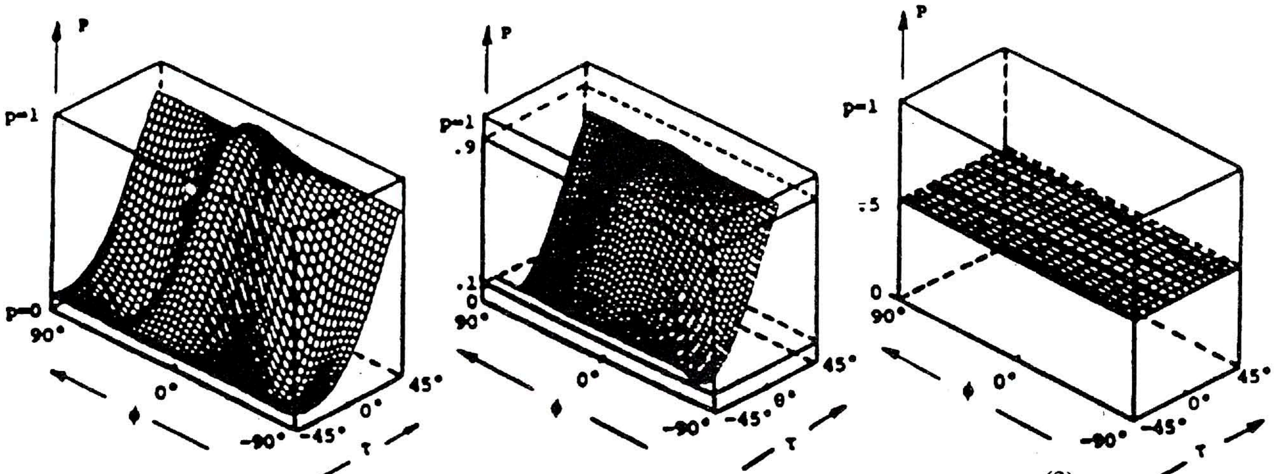


Fig. 5 - OPTIMAL POLARIZATION CHARACTERISTICS FOR PARTIALLY POLARIZED CASE: (a) Polarization dependence of the adjustable intensity in terms of the tilt and ellipticity angles; (b) Dependence of received power density plots on degree of polarization q : (1) q = 1 , (2) q = 0.8 , (3) q = 0 for the partially polarized case.

TABLE 1 - Solutions of Three Methods for the Example of $[S] = \begin{matrix} 2j & 0.5 \\ 0.5 & -j \end{matrix}$

	new basis (AB)		old basis (HV)									
	ρ'		ρ		ν'	$\vec{g}$				power	
	$\lvert\rho\rvert$	δ^0	$\lvert\rho\rvert$	δ^0	ν'	g_0	g_1	g_2	g_3	P_x	P_c
ρ_{xn1}	0	arb.	0.4142	90.0	0.00	1.0	0.7071	0.0000	0.7071	0	4.871
ρ_{xn2}	∞	arb.	2.4142	-90.0	.0	1.0	-0.7071	0.0000	-0.7071	0	0.629
ρ_{xm1}	1.0000	90	1.0000	0.0	4.5	1.0	0.0000	-1.0000	0.0000	2.25	0.50
ρ_{xm2}	1.0000	-90	1.0000	180.0	4.5	1.0	0.0000	-1.0000	0.0000	2.25	0.50
ρ_{xs1}	1.000	0	0.4142	-90.0	1.0	1.0	0.7071	0.0000	-0.7071	0.50	2.25
ρ_{xs2}	1.0000	-180	2.4142	90.0	1.0	1.0	-0.7071	0.0000	0.7071	0.50	2.25
ρ_{cn1}	1.6684	90	1.4142	-20.7	0.5	1.0	-0.3333	-0.8819	-0.3333	1.75	0
ρ_{cn2}	1.6684	90	1.4142	-20.7	0.5	1.0	-0.3333	-0.8819	-0.3333	1.75	0

TABLE 2 - Comparison of the different approaches for determining the optimal polarization states in coherent monostatic radar polarimetry

Channel			Reference							
			[7]	[6]	[8]	[9]	[10]	[5]	[19]	[1,2]
co-pol	max	$\vec{g}$	same	same	same	same	same	same	same	same
		$\vec{g}$	same	same	same	same	same	same	same	same
	null	$\vec{g}$	same	same	same	same	same	---	same	same
		$\vec{g}$	same	same	same	same	same	---	same	same
x-pol	max	$\vec{g}$	same	same	same	same	same	---	---	---
		$\vec{g}$	same	same	same	same	same	---	---	---
	saddle	$\vec{g}$	same	---	same	same	same	---	---	---
	point	$\vec{g}$	same	---	same	same	same	---	---	---
	null	$\vec{g}$	same	same	same	same	same	---	---	same
		$\vec{g}$	same	same	same	---	same	---	---	same

In extension of previous results it was found that there exist eight distinct characteristic polarization states for the symmetric matrix case, the three pairs of orthogonal polarization states whose diameters are mutually at right angles on the polarization sphere: the x-pol null pair (identical to co-pol max pair), the x-pol max pair and the x-pol saddle (turning point) pair. In addition, there exists a pair of co-pol nulls lying in the plane spanned by the x-pol-null and the x-pol max pairs, the target characteristic plane spanned by the x-pol-null and the x-pol max pairs, the target characteristic plane with the line (diameter) joining the two x-pol nulls bisecting the angle between the two co-pol nulls on this target characteristic circle. As a result of these unique polarization fork properties, one can show that once the two co-pol nulls have been found, the entire polarization fork can be recovered; i.e., for the description of a radar target we require the specification of two distinct points on the polarization sphere, whereas, only one for the description of a completely polarized wave. In particular, our polarization transformation ratio formulation is in complete agreement with Huynen's formulation and shows, given a measured matrix [S], that the Huynen target characteristic parameters m, $\phi_\mu,\nu,\gamma,\delta_m$ and α_m, can be uniquely determined; or inversely, given these parameters the scattering matrix [S] can be uniquely reconstructed (Boerner, Xi, 1990-92). Hence, the resulting Huynen fork concept represents a unique example of a fundamental polarimetric radar inverse problem.

6. OPTIMAL POLARIMETRIC CONTRAST ENHANCEMENT COEFFICIENTS: 'OPCEC'

Next to determining the eigenvalue and optimization problems for $[S(AB)]$, $[G(AB)]$, $[\Sigma(AB)]$ and $[M]$ and its optimal (characteristic) polarization states " a formidable still not completely resolved problem for either symmetric or definitely for the asymmetric cases " equally important, the exact and correct expressions for the enhancement of the optimal contrast between two classes of scatterers or scatterer ensembles must be determined. This specific optimization problem was first considered in depth by Russian and Ukranian radar polarimetrists, and we refer to the recent review by Kozlov et al. (Zozlov, Logvin, Zhivotovsky, 1992) in Boerner et al. (Boerner, 1992). In general, these two distinct classes of scatterers may be defined as 'T' and 'C', where 'T' defines, for example, the desirable (useful) scatterer (target: 'T') and 'C' the undesirable scatterer ensemble (clutter: 'C') against which 'T' is to be discriminated or to be contrasted. The formal development of these **'opcec'** expressions associated with a specific matrix description in terms of either $[S(AB)]$, $[G(AB)]$, $[\Sigma(AB)]$, $[M]$ and/or any combination of such, is also still unresolved, yet solutions are in need for introducing more meaningful and polarimetrically unique definitions for the polarimetric co/cross-polar 'signal-to-clutter ratio',co/cross-polar detection merit factors, etc. In the following, some of these **'opcec'** expressions are introduced for the separate cases of 'a priori' knowledge on $[S(AB)]$, $[G(AB)]$, $[\Sigma(AB)]$, and/or $[M]$, where in most cases unique **'opcec'** expressions for the mixed co/cross-polar power density and or relative phase coefficient problems must still be found.

6.1 OPCEC for $P_{c/x}(\rho)$ given $[S(AB)]$ for T and C: 'opcec [S]'

Several distinct solutions for either the co/co, co/cross, cross/co, cross/cross power density 'T' versus 'C' optimization cases exist, where

$$opcec\{[S]\} = \frac{P_{c/x}([S(AB)_T])}{P_{c/x}([S(AB)_C])} = \frac{\vec{h}^{\,T}_{A,B}[S(AB)_T]\vec{E}_T}{\vec{h}^{\,T}_{A,B}[S(AB)_C]\vec{E}_T} \tag{27a}$$

The solution is obtained from using the Lagarange multipliers method, and it is strongly dependent on the solution of the 'point scatterer' polarization fork solution (Boerner, Liu, Zhang, 1992; Boerner, Yan, Xi, Yamaguchi, 1992).

6.2 OPCEC for $P_{c/x}(\rho)$ given [G(AB)] for T and C: 'opcec [G]'

Also this solution [5] depends, in general, on the polarization fork solution, using the Lagrange multipliers method for solution

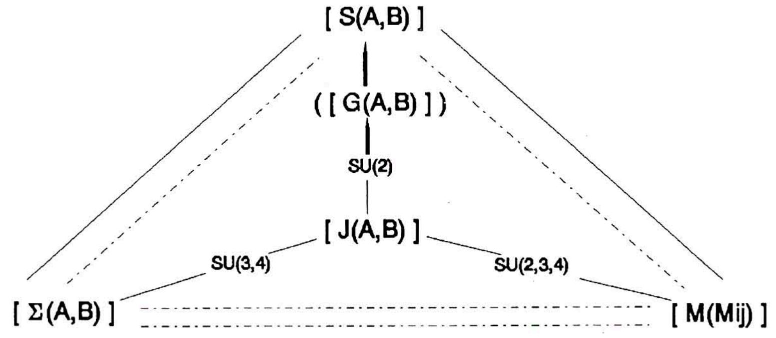
$$opcec\{[G]\} = \frac{P_{C/X/T}([G(AB)_T])}{P_{C/X/T}([G(AB)_C])} = \frac{\vec{e}^{\,+}_{X/C/T}[G_T]\vec{E}_T}{\vec{e}^{\,+}_{X/C/T}[G_C]\vec{E}_T} \tag{27b}$$

6.3 OPCEC for $P_{c/x}(\rho)$ and $P_{c_\perp}$ given $[\Sigma(AB)]$ for T and C: 'opcec $\{[\Sigma(P_i)]\}$'

From inspection of the definitions of $[\Sigma(AB)]$ of (9a) and $[\Sigma(\rho^\perp)]$ of (9b), it is apparent that in general, a distinct combination of optimal contrast enhancement relations between two scatterer classes 'T' and 'C' exists, involving either $P_c(T)$ versus $P_c^\perp(C)$ or $P_c(C)$, $P_x(C)$; $P_x(T)$ versus $P_c(C)$, $P_c^\perp(C)$, $P_x(C)$; or versus its complex conjugate, etc., and similar expressions can be found for $R_c(\rho)$, $R_x(\rho)$, etc., depending on the specific nature of $[\Sigma(AB)_T]$ and $[\Sigma(AB)_C]$. Little, yet is known, and the solutions for optimizing $[M_T]$ versus $[M_C]$ must first be established [10] in order to interpret the solutions for these cases.

6.4 OPCEC for $P_{c/x}$ given $[M]$: "opcec $\{[M_i]\}$"

In general, a partially coherent wave $\vec{g}$ can be decomposed according to (5a) into its completely polarized component $\vec{g}_q$ and unpolarized component $\vec{g}_u$, and it is the total polarized energy of the desired scatterer 'T' which is to be optimized by minimizing the respective power contribution of the undesirable scatterers 'C'. Again, several meaningful distinct opcec $[M_i]$ may be defined (Ioannidis, Hammers, 1979; Tanaka, Boerner, 1992) depending strongly on the particular nature of the scattering scenario under investigation. The solution of this rather complex multiparameter polarimetric optimization problem depends strongly on that for finding a complete set of solutions for the single scatterer solution of $[M]$ and $[\Sigma]$, and the opcec solutions for $[\Sigma(AB)]$. Here one of many possible distinct opcec definitions developed in (Tanaka, Boerner, 1992) is introduced, assuming that $[M_T]$ and M_C are known and the ratio of the completely polarized components $(g_0^q)_T$ is to be optimized versus $(g_0^q)_c$ such that



SU(2) : PAULI SPIN MATRICES

$$[\sigma_0] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad [\sigma_1] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad [\sigma_2] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad [\sigma_3] = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}$$

SU(3) : ALTERNATE GELL-MANN MATRICES (CLOUDE'S SET) :

$$[\delta_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [\delta_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad [\delta_3] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad [\delta_4] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad [\delta_5] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[\delta_6] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [\delta_7] = \begin{bmatrix} 0 & j & 0 \\ -j & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [\delta_8] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & j \\ 0 & -j & 0 \end{bmatrix} \quad [\delta_9] = \begin{bmatrix} 0 & 0 & j \\ 0 & 0 & 0 \\ -j & 0 & 0 \end{bmatrix}$$

SU(4) : DIRAC MATRICES

$$[\theta_1] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{bmatrix} \quad [\theta_2] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \\ 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{bmatrix} \quad [\theta_3] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad [\theta_4] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix} \quad [\theta_5] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$[\theta_6] = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix} \quad [\theta_7] = \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \\ -i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad [\theta_8] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \\ 1 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix} \quad [\theta_9] = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix} \quad [\theta_{10}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$[\theta_{11}] = \begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad [\theta_{12}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad [\theta_{13}] = \begin{bmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 1 \\ i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad [\theta_{14}] = \begin{bmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad [\theta_{15}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig. 6 - THE POLARIMETRIC SCATTERING MATRIX TRYPTYCH (2'2 Sinclair [S(A,B)], 2×2 Graves [G(A,B)], 3×3(4×4) Covariance [Σ(A,B)], 4×4 Mueller (propagation) / Stokes(reflection) / Kennaugh(scattering) [M(M_{ij})] matrices for symmetric AB ≠ BA (asymmetric AB c BA) cases with the SU(n) Lie groups: SU(2) 2×2 Pauli [$\sigma_i; i = 0,1,2,3$]; SU(3) → 3 × 3 Hausdorff (Gell - Mann) [δ_i; $i = 1,2,\ldots,8$]; SU(4) → 4 ×4 Dirac [θ_i; $i = 0,1,\ldots,15$] matrices).

$$opcec\left\{[\,M\,(g_0^q)\,]\right\} = \frac{(g_0^q)_T}{(g_0^q)_C} = \sqrt{\frac{(g_{T1}^2 + g_{T2}^2 + g_{T3}^2)}{(g_{C1}^2 + g_{C2}^2 + g_{C3}^2)}} =$$

$$\sqrt{\frac{\vec{g_T}\,[\,\overline{M}_T\,]^T\,[\,\overline{M}_T\,]\,\vec{g_T}}{\vec{g}\,[\,\overline{M}_C\,]^T\,[\,\overline{M}_C\,]\,g_T}} \qquad (27c)$$

with [M] denoting a $i \times j$ subset of $[\,M\,]$ where ($[\,\overline{M}\,]_{ij}$; i = 1,2,3; j = 0,1,2,3), etc. Various solutions are considered in (van Zyl, 1986, van Zyl, Papas, Elachi, 1987, Zebker, van Zyl, 1987-91) using the Lagrange multipliers method.

6.5 Unresolved Polarimetric Contrast Enhancement Optimization Problems

Whereas for the coherent point scatterer cases, the optimization problems for the contrast enhancement between two scatterers are straight-forward, this is absolutely not so far the partially coherent case for which strictly the Mueller matrices need to be optimized for the sub-millimeter wave to optical spectral regions. However, in case the co/cross-polar phases can be recovered from dual polarization coherent radar transmit/receive systems, or from multiple transmit/receive coherent polarization radar systems, the implementation of the covariance matrix approach becomes feasible simplifying the Polarimetric Contrast Enhancement Optimization problem considerably as is shown in various contributions to Boerner et al. (Boerner, 1992), and the Corrected Polarimetric Covariance Matrix presentation will soon play a key role in POL-RAD/SAR vector signal/tensor image processing within the microwave to sub-millimeter wave spectral regions. However, in LIDAR POLARIMETRY, currently we still need to implement the complete stochostic Mueller matrix optimization analysis, i.e., the complete partially coherent treatment, because 'phase correlation' of two orthogonal laser channels is technologically still not completely feasible.

7. Lie SU(n = 2,3,4) GROUP EXPANSION OF THE SCATTERING MATRICES

Although it was shown that identical solutions for the 'degenerate coherent case' are obtained from the combined eigenvalue and optimization problems of the four distinct scattering matrices $[\,S\,(AB)\,]$,$[\,G\,(AB)\,]$,$[\,\Sigma\,(AB)\,]$ and $[\,\overline{M}\,]$, no complete solutions for the partially polarized and especially the partially coherent cases, have yet been exhausted. However, well advancing feasibility studies show that such complete solutions exist and can be obtained via a reformulation of the four basic polarimetric matrices in terms of the coherence matrix $[\,J\,]$ (or coherence vector $\vec{J}$) formulation (5a) implementing SU(n = 2,3,4) Lie group expansions. Cloude (Cloude, 1986-92-90-91-88) first introduced this concept in radar polarimetry which is further expanded here in form of a matrix tryptych relating $[\,S\,(AB)]$,$[\,G\,(AB)\,]$,$[\,\Sigma\,(AB)\,]$ and $[\,M\,(AB)\,]$, with $[\,G\,(AB)\,]$ representing a power density subset of [S(AB)], as illustrated in Fig.6. Use is made of the expression of the coherency matrix $[\,J\,]$ in terms of the SU(2) group 2×2 Pauli matrix $[\,\sigma_i\,]$ and its related SU(3) 3×3 Hausdorff (or alternate Gell-Mann) $[\,E\,\delta_i\,]$, and SU(4) Dirac 4×4 $[\,\theta_i\,]$ matrices, which are listed with Fig.6.

The three sets of Lie groups SU(n = 2,3,4) are useful in reexpressing the properties of the characteristic (optimal) polarization state theories derived from the scattering matrices $[\,S\,(AB)\,]$,$[\,G\,(AB)\,]$,$[\,\Sigma\,(AB)\,]$ and $[\,M\,]$ via the expansion for the associated compact form, e.g. Huynen's polarization fork representation (16). The exponential matrix operations are derived from the general matrix exponential series expansion

$$\exp\left\{[\,A\,]\right\} = [\,I\,] + [\,A\,] + \left(\frac{1}{2!}\right)[\,A\,]^2 + \left(\frac{1}{3!}\right)[\,A\,]^3 + \ldots + \left(\frac{1}{n!}\right)[\,A\,]^n + \ldots \qquad (28)$$

together with the Campbell - Baker - Hausdorff identities

$$\exp\left\{[\,C\,]\right\} = \exp\left\{[\,A\,]\right\}\exp\left\{[\,B\,]\right\} = [\,A\,] + [\,B\,] + \frac{1}{2}[[A\,],[B\,]\,] + \frac{1}{2}[\,[\,[A\,],[\,[A\,],[B\,]\,]\,] + [\,[B\,],[B\,],[A\,]\,]\,] + \ldots \qquad (29)$$

with $[\,[\,A\,],[\,B\,]\,] = [\,A\,]\,[\,B\,] - [\,B\,]\,[\,A\,]$

$$\exp\left\{[\,S\,]\,[\,A\,]\,[\,S\,]^{-1}\right\} = [\,S\,]\exp\left\{[\,A\,]\right\}[\,S\,]^{-1}$$

$$\det\left\{\exp\left\{[\,A\,]\right\}\right\} = \exp\left\{Trace\left\{[\,A\,]\right\}\right\}$$

$$\exp\left\{[\,A\,]\right\}^{-1} = \exp\left\{-[\,A\,]\right\} \qquad (30)$$

In radar polarimetry the four classes of scattering matrices $[\,S\,]$,$[\,G\,]$,$[\,\Sigma\,]$ and $[\,M\,]$ need to be expanded in terms of the ' reduced characteristic state matrices': $[\,\psi_i\,]$ satisfying

$$[\,A\,] = \sum_{i=1}^{N} \alpha_i\,[\,\Psi_i\,]\ ,\ \alpha_i = \frac{1}{2}Trace\left\{[\,A\,]\,[\,\Psi_i\,]\right\} \qquad (30)$$

Specifically, if $\exp\left\{[\,A\,]\right\}$ represents a $n \times n$ unitary matrix

$$[\,\Psi_u\,]^+ = -[\,\Psi_u\,]\ ,\ Trace\left\{[\,\Psi_u\,]\right\} = 0 \qquad (31)$$

then the set of $(n^2 - 1)$ matrices $[\psi_u]$ are the $n \times n$ anti-hermetian matrices defined by the SU(n = 2,3,4) Lie groups.

SU(n=2) : The Pauli Spin Matrices $[\sigma_i; i = 0,1,2,3]$
As shown in (6a) and (16), the Pauli spin matrices can be used to re-expand the Sinclair and Mueller matrices, where by including the idem matrix $[\sigma_0] = [I]$, it can be shown that with the $[\sigma_i]$ of Fig. 6

$$[\sigma_1]^2 = [\sigma_2]^2 = [\sigma_0], [\sigma_1][\sigma_2] = -[\sigma_2][\sigma_1] = [\sigma_3] \quad (32)$$

so that with

$$\vec{g} = [A(HV)]\vec{J}(HV), \quad (33a)$$

the Stokes parameters can be reexpressed as

$$g_\mu = \frac{1}{2} Trace\{[\sigma_\mu] \sum_{\nu=0}^{3} [\sigma_\nu] g_\nu\} = Trace\{[\sigma_\mu][J^-(HV)]\} \quad (33b)$$

and the elements $M_{\mu\nu}$ of $[M]$ as

$$M_{\mu\nu} = (1/2)\, Trace([\sigma_\mu][S(HV)][\sigma_\nu][S(HV)]^+) . \quad (33c)$$

providing part of the tryptych solution defined in Fig. 6.

SU(n=3,4): The Gell-Mann $[\delta_{ii}; i = 1,2,\ldots,9]$ **and the Direc** $[\theta_{ii}; i = 0,1,2,\ldots,15]$ **Matrices**
Other expansions of $[\Sigma(AB = BA)]$ and of the related symmetric Mueller matrix $[M(ij = ji)]$ in terms of alternate 3 x 3 Gell-Mann matrices $[\delta_i; i = 1,2,\ldots,9]$ were first used in radar polarimetry by Cloude (Cloude, 1986-92-90-91-88) and in terms of the Dirac matrices $[\theta_i; i = 0,1,2,\ldots,15]$ by Wanielik (Wanielik, 1988); and in (Boerner, Liu, Zhang, 1992) a more complete comparative formulation is developed.In order to complete the search for the eigenvalue and optimization solutions of $[\Sigma(AB)]$ and $[M]$ in narrowband radar polarimetry, its closed from compact expressions in terms of the SU(n = 2,3,4) Lie group expansions need to be derived next, similar to Huynen's polarization fork formulation in terms of the Pauli matrices $[\sigma_i; i = 0,1,2,3]$, for both the symmetric (AB = BA/ij = ji) and the asymmetric (AB ≠ BA / ij ≠ ji) scattering matrix cases in terms of the 3 x 3 Gell - Mann and the 4 x 4 Dirac matrices, listed with Fig.6, for the symmetric and the asymmetric cases, respectively. Based on these complete closed form compact solutions, in a second step, the pertinent optimal polarimetric contrast enhancement coefficients 'opcec' for determining the 'optimal contrast' between two classes of scatterers or scatterer ensembles can be fully developed in compact closed form also for the partially coherent case.

CONCLUSION

Even so the full narrowband solutions (including the coherent, partially polarized and partially coherent cases) of the characteristic polarization states and of the associated optimal stochasticity coefficients have been in parts and may be completely determined, the quest for detecting and discriminating low RCS targets embedded in a rapidly changing, dynamic background clutter environment, for example, such as that of low RCS objects skimming over a dynamically rough, rapidly changing sea or continental tree-covered rugged terrain surface, will ultimately require the generalization of these concepts to the wideband spectral domain covering the entire ultrawideband electromagnetic non-invasive spectral region.Thus the ultimate goal is to develop tools derived in narrowband radar polarimetry which are applicable to UWB (ultrawideband) sensing and imaging of low RCS radar targets embedded in a dynamic, rapidly changing background clutter. This requires, in the next step, the generalization of the radar (cross section) scattering matrices in the frequency domain expressed in terms of the polarimetric target eigenresonance structure of the various polarization-dependent matrix elements leading to the concept of polarimetric wavelets. These concepts are being actively pursued and it can already be demonstrated that the SU(n = 2,3,4) Lie group expansions will play a dominant role in developing the most robust optimal target versus clutter contrast enhancement algorithms which are completely polarimetric ultrawideband in nature and make full use of the "polarimetric wavelet" descriptions. Whereas the narrowband polarimetric radar optimization algorithms here developed are of immediate and direct interest to the proper interpretation of narrowband POL-RAD/SAR microwave signatures in air-borne and space remote sensing in wide-area surveillance of the terrestrial and planetary environments, the development of generalized polarimetric impulse radar optimization algorithms will become essential for implementing UWB-POL-RAD/SAR sensing and imaging techniques for instantaneous detection of low RCS target embedded in dynamic background clutter in practice (Boerner, Liu, Zhang, Naik, 1992).These ultrawideband polarimetric radar problems will be the subject of a third NATO-ARW-WPDR'93 on 'Wideband Polarimetric Doppler Radar / Lidar Sensing and Imaging' as proposed in (Boerner, 1992).

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